11

MATRICES & DETERMINANTS

CONTENTS

MATRICES

Consider a firm which produces two types of goods, Ice Cream and Yoghurt, which it sells to three customers, James Bond, Harry Porter and Lionel Messi. The weekly sales for these goods are given in Table 11.1. In a particular week, the firm sells 5 barrels of Ice Cream to James Bond and 9 barrels to Harry Porter, and so on.

Table 11.1 Weekly Sales

		Ice Cream	Yoghurt
Sold to customer	James Bond	5	2
	Harry Porter	9	1
	Lionel Messi	8	6

If we ignore the table headings, this information can be written more concisely in matrix form as

$$W = \begin{bmatrix} 5 & 2 \\ 9 & 1 \\ 8 & 6 \end{bmatrix}$$

In general, a **matrix** is a rectangular array of numbers, or expressions, arranged in rows and columns. The number of rows and columns of a matrix is called its **order** or its **dimension**. For example, the dimension (or order) of the matrix below is 3 × 2 (read "three by two"), because there are three rows and two columns:

$$\begin{bmatrix} 1 & 4 \\ -9 & 2 \\ 3 & 7 \end{bmatrix}$$

The rows are horizontal and the columns are vertical. Each element of a matrix is often denoted by a variable with two subscripts. For example, a_{31} represents the element at the third row and first column of matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix Operations

Addition, subtraction, and multiplication can be performed using matrices.

Addition and Subtraction

If they have the same dimensions, two matrices can be added or subtracted element by element.

☑ Example 11.1

Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -1 \\ 5 & -2 \end{bmatrix}$. Find a) $A + B$ b) $A - B$

Solution **tips**

a) Add the corresponding elements

$$A + B = \begin{bmatrix} 1+0 & 2+-1 \\ 3+5 & 4+-2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 8 & 2 \end{bmatrix}$$

b) Subtract the corresponding elements

$$A - B = \begin{bmatrix} 1-0 & 2--1 \\ 3-5 & 4--2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

Scalar Multiplication

Scalar multiplication can also be done on any matrix by multiplying each element of the matrix by a number.

☑ Example 11.2

Given $A = \begin{bmatrix} -1 & 2 \\ 4 & 0 \\ 10 & -6 \end{bmatrix}$, find a) $2A$ b) $-3A/2$

Solution **tips**

a) Multiply each element of the matrix by 2.

$$2A = 2\begin{bmatrix} -1 & 2 \\ 4 & 0 \\ 10 & -6 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 8 & 0 \\ 20 & -12 \end{bmatrix}$$

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