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MATRICES & SIMULTANEOUS EQUATIONS

CONTENTS

INVERSE MATRIX

The simultaneous equations $a_{11}x + a_{12}y = b_1$, $a_{21}x + a_{22}y = b_2$ can be written in matrix form as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

In reduced form as: $Ak = B$

If $Ak = B$, then

$$\boldsymbol{k = A^{-1}B}$$

A is the matrix; k is the x, y, z (etc) values; and B is the constants.

☑ EXAMPLE 12.1

Solve the simultaneous equations: $-x + 2y = -5, \quad 2x - 3y = 9$

SOLUTION**tips**

Rewrite the equations in matrices form

$$\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 9 \end{bmatrix}$$

If $Ak = B$, then $k = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -5 \\ 9 \end{bmatrix} = \frac{1}{-1(-3) - 2(2)} \begin{bmatrix} -3 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -5 \\ 9 \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} -3(-5) + -2(9) \\ -2(-5) + -1(9) \end{bmatrix} = -\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$x = 3, \quad y = -1$

☑ EXAMPLE 12.2

Solve the simultaneous equations: $0.4x + 0.3y = 1, \quad -0.2x + 0.1y = 2$

SOLUTION tips

Rewrite the equations in matrices form

$$\begin{bmatrix} 0.4 & 0.3 \\ -0.2 & 0.1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

If $Ak = B$, then $k = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{0.1}\begin{bmatrix} 0.1 & -0.3 \\ 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}$$

$x = -5, \quad y = 10$

☑ EXAMPLE 12.3

Solve the simultaneous equations using inverse matrix:
$2x + 3y - z = 4$, $3x + 4y + 2z = 2$, and $-y + 5z = -6$.

SOLUTION tips

Rewrite the equations in matrix form: $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 4 & 2 \\ 0 & -1 & 5 \end{bmatrix}$

$$|A| = +2\begin{vmatrix} 4 & 2 \\ -1 & 5 \end{vmatrix} - 3\begin{vmatrix} 3 & 2 \\ 0 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 4 \\ 0 & -1 \end{vmatrix}$$
$$= 2(20 - (-2)) - 3(15 - 0) - 1(-3 - 0)$$
$$= 44 - 45 + 3 = 2$$

$$C = \begin{bmatrix} +\begin{vmatrix} 4 & 2 \\ -1 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 0 & 5 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix} \\ +\begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (20 - (-2)) & -(15 - 0) & (-3 - 0) \\ -(15 - 1) & (10 - 0) & -(-2 - 0) \\ (6 - (-4)) & -(4 - (-3)) & (8 - 9) \end{bmatrix} = \begin{bmatrix} 22 & -15 & -3 \\ -14 & 10 & 2 \\ 10 & -7 & -1 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} 22 & -14 & 10 \\ -15 & 10 & -7 \\ -3 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A = \frac{1}{2}\begin{bmatrix} 22 & -14 & 10 \\ -15 & 10 & -7 \\ -3 & 2 & -1 \end{bmatrix}$$

If $Ak = B$, then $k = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 22 & -14 & 10 \\ -15 & 10 & -7 \\ -3 & 2 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 22(4) - 14(2) + 10(-6) \\ -15(4) + 10(2) - 7(-6) \\ -3(4) + 2(2) - 1(-6) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$x = 0, y = 1, z = -1$

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