16

LINEAR PROGRAMMING

CONTENTS

A LINEAR PROGRAMMING PROBLEM

Linear Programming (LP) is an optimization technique used to maximize or minimize a linear function (usually profit or cost of production), subject to a set of constraints. It was developed by Leonid Kantorovich around the time of WWII. LP is a special case of **mathematical programming** (also called mathematical optimization). Mathematical Programming is the use of mathematical models in the optimum allocation of limited resources among competing activities, under a set of constraints. There are well-known successful applications in engineering (for shape optimisation), efficient manufacturing (for maximisation of profit), energy industry (for optimisation of the electric power system), and transportation optimisation (for cost and time efficiency).

The standard form of a LP problem consists of the following three parts:

A linear function to be maximized/minimized

e.g. $Z = c_1x_1 + c_2x_2$

Problem constraints of the following form

e.g. $$\begin{aligned} a_{11}x_1 + a_{12}x_2 &\le b_1 \\ a_{21}x_1 + a_{22}x_2 &\le b_2 \end{aligned}$$

Non-negativity restrictions

e.g. $x_1 \ge 0, x_2 \ge 0$

where the **objective function,** Z, is the expression to be maximized or minimized; *x* represents the variables to be determined; *a*, *c* and *b* are constants. The **inequalities** are the constraints over which the objective function is to be optimized. The **non-negativity restriction** of all the variables is a condition always implicit in the methods of solution in LP.

Steps in Solving a LP problem

If the problem is not a word/story problem, skip to step 5.

1. Identify the decision variables (e.g., $x_1,\ x_2$). **Decision variables** describe the quantities that the decision makers would like to determine. They are the unknowns of a LP problem.
2. Write the objective function represented by Z. The **objective function** is the function whose value is to be either minimized or maximized (e.g., $Z = x_1 + x_2$).
3. State the constraints (e.g., $x_1 + x_2 \geq 1$). The **constraints** are the restrictions or limitations on the decision variables.
4. Note the non-negativity restriction (e.g., $x_1 \geq 0,\ x_2 \geq 0$). **Non-negativity restriction** indicates that all decision variables must take on values equal to or greater than zero.
5. Solve the LP problem. The most popular methods are graphical, matrices and simplex method.

GRAPHICAL METHOD

If a LP problem has two decision variables, the graphical method can find the optimal solution. In this method, the inequalities are plotted in the XY graph where the intersecting region forms the **feasible region**. The feasible region provides the optimal solution.
In the graphical method, the **fundamental theorem of LP** or the **corner point theorem** is important.

> **The fundamental theorem of LP** or the **corner point theorem** states that, if a LP problem is feasible, the maximum or minimum value will occur at a corner point, or on a line segment between two corner points.

To find the optimal values in the feasible region, we test the corner points. Any corner point that provides the largest value for the objective function is the maximum; any corner point that provides the smallest value for the objective function is the minimum.

Steps in the Graphical Method:

1. Formulate the LP problem.
2. Graph the inequalities: locate the feasible region and the corner points.
3. Determine the value of the objective function at each corner point.
4. Identify the optimum point.

☑ EXAMPLE 16.1

Solve the following LP problem using the graphical method:

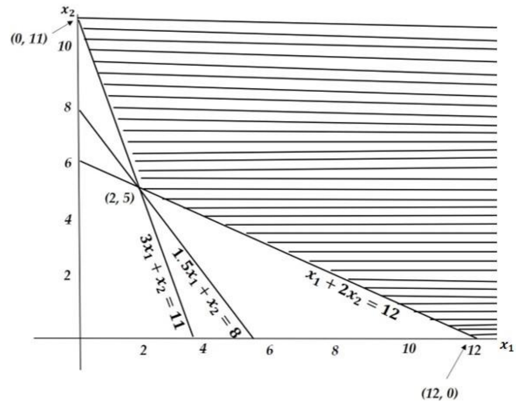
$$\text{Min } C = 2x_1 + x_2 \text{ subject to } \begin{cases} x_1 + 2x_2 \geq 12 \\ 1.5x_1 + x_2 \geq 8 \\ 3x_1 + x_2 \geq 11 \end{cases} \quad x_1 \geq 0,\ x_2 \geq 0$$

SOLUTION**tips**

First graph the inequalities. Find two solutions, preferably the x_1-intercepts and x_2-intercepts of the graph, by setting first $x_1 = 0$ and then $x_2 = 0$.

$\boldsymbol{x_1 + 2x_2 = 12}$	$\boldsymbol{1.5x_1 + x_2 = 8}$	$\boldsymbol{3x_1 + x_2 = 11}$
$x_1 = 0, x_2 = 6$	$x_1 = 0, x_2 = 8$	$x_1 = 0, x_2 = 11$
$x_2 = 0, x_1 = 12$	$x_2 = 0, x_1 = 5.33$	$x_2 = 0, x_1 = 3.67$

For each inequality, plot the two points and draw a line connecting them. The shaded area is the feasible region. The feasible region is the one which satisfies all the inequalities.



Then find the corner points of the shaded polygon formed. A corner point is a point in the feasible region that is the intersection of two boundary lines. Substitute the corner points into the objective function (C) and find the smallest value.

Corner Point	$C = 2x_1 + x_2$
(0, 11)	$2(0) + (11) = 11$
(2, 5)	$\mathbf{2(2) + (5) = 9}$
(12, 0)	$2(12) + (0) = 24$

The minimum value is 9 at (2, 5).
Hence, the optimal solution to the given LP problem is: $x_1 = 2$, $x_2 = 5$ and $C = 9$.

☑ EXAMPLE 16.2

Fitness Engineering produces only two fitness trackers: Grandeur and Posh. Production is limited to 20 hours per week, and the production of Grandeur requires 2 hours while the production of Posh requires 1 hour. The firm has only 16 cubic feet of storage space and a package of Grandeur requires 1 cubic feet, while a package of Posh requires 1 cubic feet. The demand is such that no more than 8 packages of Grandeur can be sold each week. The management has determined that the profit are \$2.05 for each Grandeur and \$1.05 for each Posh.

How many fitness trackers of each type should be produced each week in order to maximize the total profit?

SOLUTION**tips**

First define the decision variables and the units in which they are measured. For this problem we are interested in knowing the optimal number of packages of each type of fitness tracker to produce per week.

Let $x_1 =$ Number of packages of Grandeur produced per week.
$x_2 =$ Number of packages of Posh produced per week.

The objective function is easy to identify since the idea is to maximize the profit: The management has determined that the profit are \$2.05 for each Grandeur and \$1.05 for each Posh. Which is given by:

$$Z = 2.05x_1 + 1.05x_2$$

Production is limited to 20 hours per week, and production of Grandeur requires 2 hours while production of Posh requires 1 hour. Now, the constraint imposed on production capacity per week is:

$$2x_1 + x_2 \leq 20$$

The firm has only 16 cubic feet of storage space and a package of Grandeur requires 1 cubic feet, while a package of Posh requires 1 cubic feet. Now, the constraint on storage capacity per week is:

$$x_1 + x_2 \leq 16$$

The demand is such that no more than 8 packages of Grandeur can be sold each week. Now, the constraint on demand is:

$$x_1 \leq 8$$

Since the decision variables must be nonnegative,

$$x_1 \geq 0, \ x_2 \geq 0$$

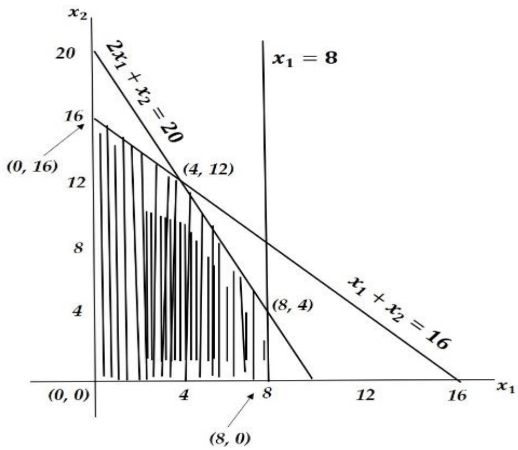
Thus, the LP is:

$$\text{Max } Z = 2.05x_1 + 1.05x_2 \text{ subject to } \begin{cases} 2x_1 + x_2 \leq 20 \\ x_1 + x_2 \leq 16 \\ x_1 \leq 8 \end{cases} \quad x_1 \geq 0, \ x_2 \geq 0$$

First graph the inequalities. For the first two inequalities, find two solutions, preferably the x_1-intercepts and x_2-intercepts of the graph, by setting first $x_1 = 0$ and then $x_2 = 0$. For the third inequality, the graph is the horizontal line: $x_1 = 8$.

$\mathbf{2x_1 + x_2 = 20}$	$\mathbf{x_1 + x_2 = 16}$	$\mathbf{x_1 = 8}$
$x_1 = 0, x_2 = 20$	$x_1 = 0, x_2 = 16$	Treat as $x_1 = 8$
$x_2 = 0, x_1 = 10$	$x_2 = 0, x_1 = 16$	Here the line is parallel to Y-axis.

For each inequality, plot the two points and draw a line connecting them. The shaded area is the feasible region.



Then find the corner points of the shaded polygon formed. Substitute the corner points into the objective function (the Z function) and find the smallest value.

Corner Point	$Z = 2.05x_1 + 1.05x_2$
(0, 0)	$2.05(0) + 1.05(0) = 0$
(8, 0)	$2.05(8) + 1.05(0) = 16.4$
(8, 4)	$2.05(8) + 1.05(4) = 20.6$
(4, 12)	$\mathbf{2.05(4) + 1.05(12) = 20.8}$
(0, 16)	$2.05(0) + 1.05(16) = 16.8$

The maximum value of the objective function $Z = 20.8$ at (4, 12).
Hence, 4 Grandeur and 12 Posh should be produced per week in order to obtain the maximum profit of $20.80.

☑ EXAMPLE 16.3

Mach100 Ltd makes two products X and Y, and has a total production capacity of 4.5 tons per week. The firm has a contract to supply at least 1 ton of X and at least 0.5 ton of Y per week. In production, each ton of X requires 4 machine hours and each ton of Y requires 12 machine hours. The available number of machine hours is 38. If the profit is $4 per ton of X and $6 per ton of Y, determine the production schedule that yields the maximum profit.

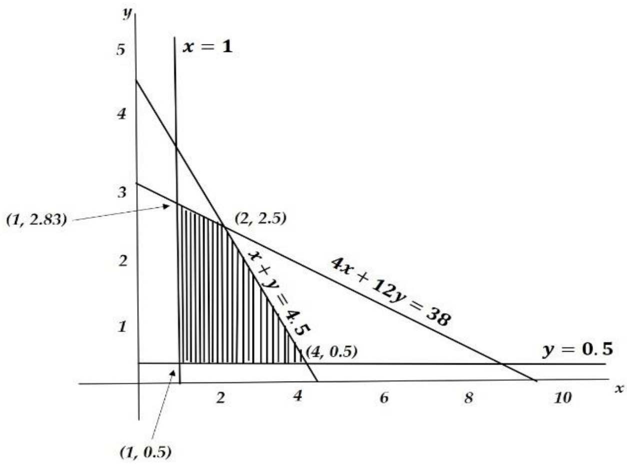
SOLUTION **tips**

Let $x =$ Number of tons of X produced per week.
$y =$ Number of tons of Y produced per week.

Thus, the LP is:
Max $Z = 4x + 6y$

$$\text{subject to} \begin{cases} x + y \leq 4.5 & \text{Production capacity} \\ x \geq 1;\ \ y \geq 0.5 & \text{Supply} \\ 4x + 12y \leq 38 & \text{Machine hours} \end{cases} \qquad x \geq 0,\ \ y \geq 0$$

$\boldsymbol{x + y = 4.5}$	$\boldsymbol{4x + 12y = 38}$
$x = 0, y = 4.5$	$x_1 = 0, y = 3.17$
$y = 0, x = 4.5$	$y = 0, x = 9.5$



Corner Point	$C = 4x_1 + 6x_2$
(1, 0.5)	$4(1) + 6(0.5) = 7$
(4, 0.5)	$4(4) + 6(0.5) = 19$
(2, 2.5)	$\mathbf{4(2) + 6(2.5) = 23}$
(1, 2.83)	$4(1) + 6(2.83) = 21$

The maximum value is 23 at (2, 2.5).
Hence the company should produce 2 tons of product *X* and 2.5 tons of product *Y* in order to yield a maximum profit of \$23.

☑ EXAMPLE 16.4

A global manufacturing outfit must decide on the optimal mix of two possible plants of which the inputs and outputs per production run are as follows:

Plants	Input (tons)		Output (tons)	
	A	B	X	Y
Chicago	1	0.5	1.25	2
Lagos	1	1	1	1

The maximum amounts available of input A and B are 45 tons and 30 tons, respectively. The market conditions require the production of at least 25 tons of X and 20 tons of Y. The profits per production run for the Chicago and Lagos plants are $15 and $20, respectively. Determine the production run for the Chicago and Lagos plants.

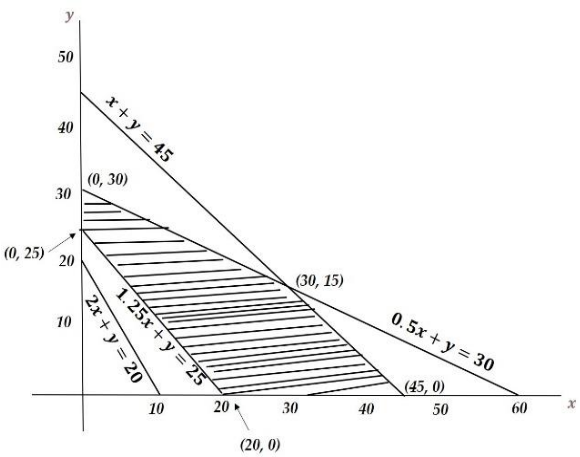
SOLUTION**tips**

Let $x =$ Number of production run for the Chicago plant.
$y =$ Number of production run of the Lagos plant.

Thus, the LP is:

$$\text{Max } Z = 15x + 20y \text{ subject to } \begin{cases} x + y \leq 45 & \text{Input} \\ 0.5x + y \leq 30 & \\ 1.25x + y \geq 25 & \text{Output} \\ 2x + y \geq 20 & \end{cases} \qquad x \geq 0, \; y \geq 0$$

$x + y = 45$	$0.5x + y = 30$	$1.25x + y = 25$	$2x + y = 20$
$x = 0, y = 45$	$x = 0, y = 30$	$x = 0, y = 25$	$x_1 = 0, y = 10$
$y = 0, x = 45$	$y = 0, x = 60$	$y = 0, x = 20$	$y = 0, x = 20$



Corner Point	$Z = 15x_1 + 20x_2$
(0, 25)	$15(0) + 20(25) = 500$
(0, 30)	$15(0) + 20(30) = 600$
(30, 15)	$\mathbf{15(30) + 20(15) = 750}$
(45, 0)	$15(45) + 20(0) = 675$
(20, 0)	$15(20) + 20(0) = 300$

The maximum value is 750 at (30, 15).
Hence, the company should produce 30 tons in Chicago and 15 tons in Lagos in order to achieve the maximum profit of $750.

↘ WORKOUT 16.1

1. Use the graphical method to solve the following LP problem:

a) Min $C = x_1 + 18x_2$ subject to $\begin{cases} x_1 + 2x_2 \geq 12 \\ 0.5x_1 + x_2 \geq 6 \\ x_1 - x_2 \geq 9 \end{cases}$

b) Max $Z = x_1 + 3x_2$ subject to $\begin{cases} x_1 + \frac{1}{2}x_2 \leq 500 \\ \frac{3}{4}x_1 + x_2 \leq 600 \\ x_1 + x_2 \leq 700 \\ x_1 - x_2 \leq 350 \end{cases}$

2. Consider i-Fashions Factory which has two designs in its garment production line—Paradise and Garden of Eden. The following table shows the scheduling hours per garment in each department and the capacities of the departments, in monthly hours.

	Paradise (hours)	Garden of Eden (hours)	Capacity (hours/month)
Purchasing	2.1	2.1	294
Manufacturing	1.575	1.05	210
Sales	1.05	2.1	210

The management has determined that the costs are $10 for each Paradise and $5 for each Garden of Eden. Find a production plan to minimize cost.

3. A workshop has three machines: Mini, Mega and Pro. It can manufacture two fragrances (Luxurious and Sumptuous) which passes through each machine in the same order: First to Mini, then to Mega and then to Pro as shown in the following table.

	Luxurious (hours/gram)	Sumptuous (hours/gram)	Capacity (hours/week)
Mini	1	4	60
Mega	5	2	48
Pro	3	1	26
Profit ($/g)	1	1	

How many of each fragrance should be produced in order that the profit is maximum?

4. Magnet Manufacturing owns plants in two cities: London and Washington. The London plant costs $500 per month to operate, and it can produce 200 barrels of A oil, 300 barrels of B oil, and 500 barrels of C oil each month. The Washington plant costs $800 per month to operate, and it can produce 100 barrels of A oil, 800 barrels of B oil, and 500 barrels of C oil each month. The company has orders totaling 1800 barrels of A oil, 11800 barrels of B oil, and 8000 barrels of C oil. How many months should each factory be run to fulfill the order on hand at minimum cost?
5. Zaz is on a diet that gives her calories, protein and carbohydrate from two food items. The diet chart is as follows:

	Food A	Food B
Calories (grams)	350	250
Protein (grams)	5	2
Carbohydrates (grams)	7	3
Cost	$5	$3

The diet has to be planned in such a way that it would contain at least 2300 calories, 25 grams of protein, and 36 grams of carbohydrates. Find a diet plan to minimize cost.

6. Giantez Global makes two types of shoes: Cruiser and Sophisticated. Each pair of Cruiser requires 5 labour hours for fabricating, 2 labour hours for finishing and 1 labour hour for sales. Each pair of Sophisticated requires 1 labour hour for fabricating, 8 labour hours for finishing and 3 labour hours for sales. For fabricating, finishing and sales, the maximum labour hours available are 44, 48 and 20 respectively. The company makes a profit of $100 on each pair of shoes. How many pairs of each type of shoes should be manufactured per week to realise a maximum profit?
7. A manufacturer must decide on the optimal mix of two production processes of which the inputs and outputs per production run are as follows:

Process	Input (tons)		Output (tons)	
	A	B	X	Y
P_1	3	1	2.5	3
P_2	3	2	2	1.5

The maximum amounts available of input A and B are 90 tons and 40 tons, respectively. The market conditions require the production of at least 50 tons of X and 30 tons of Y. The profits per production run for the P_1 and P_2 processes are $3 and $4, respectively. Determine the production run for the two processes.

ANSWERS RAPID

1. a) $x_1 = 12; x_2 = 0; C = 12$ b) $x_1 = 0; x_2 = 600; Z = 1800$
2. Paradise = 0. Eden = 200. C = 1000
3. Luxurious = 4. Sumptuous = 14. Z = 18
4. London = 2 months. Washington = 14 months. C = 12200
5. Food A = 3. Food B = 5. C = $30
6. Cruiser = 8, Sophisticated = 4. Z = 1200
7. $X = 20, Y = 10, Z = 100.

MATRIX METHOD

If the number of decision variables is equal to the number of constraints, Cramer's rule is easily applied to a LP problem. Cramer's rule is a method for solving systems of linear equations using determinants.

☑ EXAMPLE 16.5

Given the LP problem

Max $Z = 2.5x_1 + x_2 + 1.4x_3$ subject to $\begin{cases} 2x_1 + x_2 + x_3 \leq 12 \\ x_1 + x_2 + x_3 \leq 10 \\ x_1 - x_2 + x_3 \leq 4 \end{cases}$

SOLUTION**tips**

Set up the constraints as a matrix: $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 12 \\ 10 \\ 4 \end{bmatrix}$

Use Cramer's rule,

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2 \qquad |X_1| = \begin{vmatrix} 12 & 1 & 1 \\ 10 & 1 & 1 \\ 4 & -1 & 1 \end{vmatrix} = 4$$

$$|X_2| = \begin{vmatrix} 2 & 12 & 1 \\ 1 & 10 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 6 \qquad |X_3| = \begin{vmatrix} 2 & 1 & 12 \\ 1 & 1 & 10 \\ 1 & -1 & 4 \end{vmatrix} = 10$$

$$x_1 = \frac{|X_1|}{|A|} = \frac{4}{2} = 2 \qquad x_2 = \frac{|X_2|}{|A|} = \frac{6}{2} = 3 \qquad x_3 = \frac{|X_3|}{|A|} = \frac{10}{2} = 5$$

From the objective Function

$$Z = 2.5x_1 + x_2 + 1.4x_3 = 2.5(2) + (3) + 1.4(5) = 15$$

The maximum value is 15 at (2, 3, 5).

Hence, the optimal solution to the given LP problem is: $x_1 = 2$, $x_2 = 3$, $x_3 = 5$ and $Z = 15$.

☑ EXAMPLE 16.6

GX Farms has determined the best mix of fertilizers (A, B and C) to provide its crops with the desired amounts of active chemicals (nitrogen, phosphorus and potash) as shown in the following table.

	A (kg)	B (kg)	C (kg)	Minimum daily requirements (units)
Nitrogen	1	4	0	16
Phosphorus	3	8	3	92
Potash	0	2	1	18
Cost ($/kg)	2	10	1	

Determine the best mix of fertilizers that minimize cost.

SOLUTION**tips**

Let $x_1 =$ amount of fertilizer A, $x_2 =$ amount of fertilizer B, and $x_3 =$ amount of fertilizer C.

It is clear from the question that the LP minimization problem is

$$\text{Min } C = 2x_1 + 10x_2 + x_3 \text{ subject to } \begin{cases} x_1 + 4x_2 \geq 16 \\ 3x_1 + 8x_2 + 3x_3 \geq 92 \\ 2x_2 + x_3 \geq 18 \end{cases}$$

Set up the constraints as a matrix: $\begin{bmatrix} 1 & 4 & 0 \\ 3 & 8 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 16 \\ 92 \\ 18 \end{bmatrix}$

Use Cramer's rule,

$$|A| = \begin{vmatrix} 1 & 4 & 0 \\ 3 & 8 & 3 \\ 0 & 2 & 1 \end{vmatrix} = -10 \qquad |X_1| = \begin{vmatrix} 16 & 4 & 0 \\ 92 & 8 & 3 \\ 18 & 2 & 1 \end{vmatrix} = -120$$

$$|X_2| = \begin{vmatrix} 1 & 16 & 0 \\ 3 & 92 & 3 \\ 0 & 18 & 1 \end{vmatrix} = -10 \qquad |X_3| = \begin{vmatrix} 1 & 4 & 16 \\ 3 & 8 & 92 \\ 0 & 2 & 18 \end{vmatrix} = -160$$

$$x_1 = \frac{|X_1|}{|A|} = \frac{-120}{-10} = 12 \quad x_2 = \frac{|X_2|}{|A|} = \frac{-10}{-10} = 1 \quad x_3 = \frac{|X_3|}{|A|} = \frac{-160}{-10} = 16$$

The best mix of fertilizers: Fertilizer A = 12 kg, Fertilizer B = 1 kg, and Fertilizer C = 16 kg.

From the objective function:

$$\text{C} = 2x_1 + 10x_2 + x_3 = 2(12) + 10(1) + (16) = 50$$

The minimum cost at which the farm can fertilize the crops is \$50.

↘ WORKOUT 16.2

1. Given the LP problem.

   a) $\text{Min } C = x_1 + x_2 + x_3$ subject to $\begin{cases} 2x_1 + x_2 + x_3 \leq 60 \\ x_1 + x_2 + x_3 \leq 50 \\ x_1 - x_2 + x_3 \leq 20 \end{cases}$

   b) $\text{Max } Z = x_1 + 5x_2 + \frac{1}{2}x_3$ subject to $\begin{cases} 0.5x_1 + 2x_2 \leq 8 \\ 1.5x_1 + 4x_2 + 1.5x_3 \leq 46 \\ x_2 + 0.5x_3 \leq 9 \end{cases}$

2. Challenger Fashions produces only two designers: Flash and Blaze. Sewing is limited to 600 hours per month, and sewing of Flash requires 7 hours while sewing of Blaze requires 2.5 hours. Sales is \$365 per month such that no more than 1 Flash designer and 2 Blaze designers can be sold each month. The profit are \$10 for each type of designer. How many designers of each type should be produced each month in order to maximize profit?
3. King Nebuchadnezzar owns two factories which cost \$1,000 each per day to operate. Factory 1 can produce 1 barrel of A-margarine and 8 barrels of B-margarine each day. Factory 2 can produce 1.4 barrels of A-margarine and 2 barrels of B-margarine each day. The company has orders totaling 24 barrels of A-margarine and 146 barrels of B-margarine. How many days should he run each factory to minimize costs?

4. Manna Electric produces three brands of earpiece: Vax, Qax and Zax. The table shows the production schedule and costs.

	Vax (hours/unit)	Qax (hours/unit)	Zax (hours/unit)	Hours available
Fabrication	25	100	0	2000
Electronic	75	200	75	11500
Assembly	0	50	25	2250
Cost per unit	\$100	\$500	\$50	

How many of each brand should be produced in order that the production cost is minimum?

5. Consider Gorilla Inverters which has three brands in its production line—G1, G2 and G3. The following table shows the scheduling hours per inverter in each department and the capacities of the departments, in monthly hours.

	G1 (hours)	G2 (hours)	G3 (hours)	Capacity (hours/month)
Purchasing	40	160	0	6400
Manufacturing	120	320	120	36800
Sales	0	80	40	7200

The management has determined that the costs are \$10 for each G1, \$50 for each G2 and \$5 for each G3. Find a production plan to minimize cost.

ANSWERS RAPID

1. a) $x_1 = 10, x_2 = 15, x_3 = 25; C = 50$
   b) $x_1 = 12, x_2 = 1, x_3 = 16; Z = \$25$
2. a) Flash = 25, Blaze = 170, Z = \$1950
3. a) Factory 1 = 17 days. Factory 2 = 5 days. C = \$22,000
4. Vax = 60, Qax = 5 and Zax = 80. C = \$12,500
5. G1 = 120. G2 = 10. G3 = 160. C = \$2500

WORKOUT EXTRA

I. FILL IN THE BLANKS

1. LP is an .... technique used to maximize or minimize a linear function (usually profit or cost of production), subject to a set of ....
2. The inequalities are the constraints over which the .... function is to be optimized.
3. .... variables describe the quantities that the decision makers would like to determine.

II. MULTIPLE CHOICE QUESTIONS

1. Given: Min $C = 40x_1 + 30x_2$ subject to $\begin{cases} 4x_1 + x_2 \geq 2 \\ x_1 + 3x_2 \geq 3 \\ 4x_1 + x_2 \geq 3.2 \end{cases}$, which of the following is incorrect?
   a. $x_1 = 0.6$ b. $x_2 = 0.2 + x_1$ c. $C = 60x_2$ d. $x_1 = x_2$

2. The table shows the least costly mix required to produce workwear for Equilateral Factory.

	Experimental	Synthetic	Leather	Orders
A	0.5	2	0	40
B	1.5	4	1.5	230
C	0	1	0.5	45
Cost ($)	1	3	1	

If the optimal solutions are x_1, x_2 and x_3, which of the following is correct?
a. $x_1 + 4x_2 = x_3$ b. $x_1 = x_2 - 3x_3$ c. $x_2 = x_3$ d. $x_1/x_3 = 2$

3. Consider a manufacturer of android phones with two models in its production line— Grandeur and Posh. The assembly department assembles each phone from component parts produced in the mechanical and electronics departments. The following table shows the production hours per phone in each department and the capacities of the departments, in weekly hours.

	Grandeur (hours)	Posh (hours)	Capacity (hours/week)
Mechanical	3	4	60
Electronics	1	3	30
Assembly	1	1	18

The management has determined that the profit are $30 for each Grandeur and $40 for each Posh. Determine a production plan to maximize weekly profit.
a. 12, 6 b. 11, 7 c. 10, 5 d. 100, 8

4. A manufacturer produces butter and margarine. It takes 12 hours of work on machine A and 6 hours on machine B to produce a package of butter. It takes 3 hours on machine A and 4 hours on machine B to produce a package of margarine. The company earns a profit of $6 per package on butter and $3 per package on margarine. If he operates machine A and machine B for at the most 240 and 300 hours respectively per week, how many packages of each should be produced each week so as to maximise his profit?
a. 3; 75 b. 4; 77 c. 2; 72 d. 1; 76

5. Which theorem is crucial in the graphical method of solving LP problems?
   a. Corner Point Theorem
   b. Pythagorean Theorem
   c. Fundamental Theorem of Calculus
   d. Law of Sines and Cosines

ANSWERS RAPID

I	II
1. optimization; constraints 2. objective 3. decision	1. D 2. A 3. A 4. C 5. A