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LINEAR PROGRAMMING: THE SIMPLEX METHOD

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THE SIMPLEX METHOD

Simplex method was developed by George Dantzig in 1949. It is an iterative procedure for solving LP in a finite number of steps, in such a way that the value of the objective function at each step is optimized.

Steps in the simplex method:

- Convert the constraints into equations by introducing slack variables.
- Set up the initial tableau.
- Perform the pivot operation.
- Create a new tableau.
- Check for optimality. If optimal, then identify optimal values.

MAXIMIZATION

The simplex method is an effective algorithm for solving linear programming problems, especially maximization of a linear function subject to linear constraints.

☑ EXAMPLE 17.1

Musically Ltd produces only two instruments: Piano and Guitar. Both requires strings and boards only. In production, each Piano requires 1 string and 1 board while each Guitar requires 1 string and 2 boards. The company has a total of 40 strings and 75 boards. On each sale, the company makes a profit of $10 per Piano sold and $12 per Guitar sold. Now, the company wishes to maximize its profit. How many Pianos and Guitars should it produce respectively?

SOLUTION**tips**

Let x_1 equals the number of Pianos produced and x_2 the number of Guitars produced.
So, the decision variables are: x_1 and x_2.

On each sale, the company makes a profit of $10 per Piano sold and $12 per Guitar sold.
Thus, the objective function is:

$$Z = 10x_1 + 12x_2$$

In production, each Piano requires 1 unit of strings and 1 unit of boards while each Guitar requires 1 unit of strings and 2 units of boards. The company has a total of 40 units of strings and 75 units of boards.
Therefore, the constraints are

$$x_1 + x_2 \le 40$$
$$x_1 + 2x_2 \le 75$$

Number of Pianos and Guitars should be greater than or equal to 0. So, the non-negativity restriction:

$$x_1 \ge 0, x_2 \ge 0$$

Therefore, the LP problem is
Max $Z = 10x_1 + 12x_2$

$$\text{subject to} \begin{cases} x_1 + x_2 \le 40 & \text{(units of strings)} \\ x_1 + 2x_2 \le 75 & \text{(units of boards)} \end{cases} \quad x_1 \ge 0, x_2 \ge 0$$

Now, to apply the simplex method, introduce slack variables to convert the inequalities into equalities.

$$x_1 + x_2 + s_1 = 40$$
$$x_1 + 2x_2 + s_2 = 75$$

s_1 and s_2 are slack variables where $s_1 \ge 0, s_2 \ge 0$.
Slack variables represent the amount of an unused resource.

Modify the objective function so that the RHS is zero:

$$Z = 10x_1 + 12x_2 \quad \Rightarrow \quad Z - 10x_1 - 12x_2 = 0$$

Convert the system of equations into a tableau in the following format.

Tableau I

Basic	x_1	x_2↓	s_1	s_2	RHS	Ratio
s_1	1	1	1	0	40	40 / 1 = 40
s_2	1	**2**	0	1	75	75 / 2 = 37.5
Z	-10	-12	0	0	0	

Perform a pivot operation if there are negative entries in the Z row. The current tableau is only optimal if and only if every entry in the Z row is nonnegative.