

19

GAME THEORY

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Game theory, pioneered by John von Neumann and Oskar Morgenstern, is a field of applied mathematics that provides methods for studying scenarios where multiple individuals, referred to as players, make decisions that impact each other. These decisions are interdependent, meaning that players must take into account the possible choices or strategies of the other players when devising their own strategies. The goal is to find a solution that outlines the best decisions for all players involved, even if their interests align, conflict, or have a combination of both, and to determine the potential outcomes that may arise from these decisions.

Game theory has found widespread application, ranging from predicting the formation of political alliances or business conglomerates, determining the optimal pricing strategy in competitive markets, assessing the influence of individual voters or voting blocs, selecting jury members, identifying ideal locations for manufacturing facilities, examining the behavior of various animals and plants in their struggle for survival, and even challenging the legality of certain voting systems. A classic example of game theory is a hypothesis called "Prisoners Dilemma".

THE PRISONER'S DILEMMA

The Prisoner's Dilemma, a classic game theory problem, is a thought experiment where two rational agents face a dilemma: to cooperate for mutual reward, or betray (defect) for individual reward. Originally framed by Merrill

Flood and Melvin Dresher in 1950, the game was formalized and named "prisoner's dilemma" by Albert W. Tucker.

Consider the example of two members of a criminal gang arrested and imprisoned. In solitary confinement, neither prisoner has the means to communicate with each other. Prosecutors have no hard evidence to convict the pair on the principal charge. However, to gain a confession, the police offer each prisoner four deals (displayed as a 2×2 box in Figure 19.1):

1. If both remain silent, each will go to jail for only 2 years.
2. If A betrays B but B remains silent, then A will be set free while B goes to jail for ten years.
3. If A remains silent but B betrays A, then A will go to jail for 10 years while B is set free.
4. If both betrays each other, then both goes to jail for 5 years.

Figure 19.1 Prisoner's Dilemma

	B	B stays silent	B betrays
A	A stays silent	-2	0
	A betrays	-10	-5

In this game, loyalty to one's partner is irrational. This assumption of rationality suggests that the only possible outcome for two rational prisoners is betrayal, although mutual cooperation would yield a higher net reward.

The prisoner's dilemma game models many real world situations involving cooperative behavior such as oligopolistic competition and collective action to produce a collective good. For example, in the absence of legislation and penalties, members of a cartel can engage in a (multi-player) prisoner's dilemma: Cooperation would mean keeping prices at a pre-agreed minimum level while defection means selling below this minimum level, thus taking business (and profits) from other members.

ZERO-SUM GAMES

A **zero-sum game** is a situation which involves two sides, where the result is an advantage to one side and an equivalent loss to the other. Player A's gain is equivalent to player B's loss; if the total gains of the players are added up and the total losses are subtracted, they will sum to zero (see Figure 19.2). Examples of zero-sum games include bridge, chess and poker where A gains and B loses.

Figure 19.2 Example of the Classic Zero-Sum Game

	Option 1	Option 2
Option 1	3, -3	-3, 3
Option 2	-3, 3	3, -3

In contrast, a **non-zero-sum game** is a situation where the sum of gains and losses by the players is more or less than zero. A zero-sum game is a strictly competitive game

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