2

APPLICATIONS OF DIFFERENTIAL CALCULUS

CONTENTS

CONCAVITY, CONVEXITY AND RELATIVE EXTREMA

A **convex** function has an increasing first derivative, making it appear to bend upwards, as shown in Figure 2.1. Conversely, a **concave** function has a decreasing first derivative making it bend downwards. Mathematically,

$f'(a) > 0$ increasing function at $x = a$
$f'(a) < 0$ decreasing function at $x = a$

First-derivative test or first-order condition for concavity/convexity:

$f'(a) > 0$ $f(x)$ is convex at $x = a$
$f'(a) < 0$ $f(x)$ is concave at $x = a$

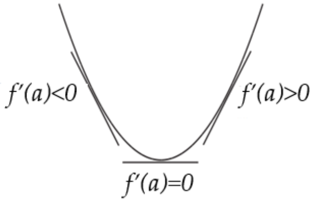
Second derivative test or second-order condition:

$f''(a) > 0$ local minimum at $x = a$
$f''(a) < 0$ local maximum at $x = a$
$f''(a) = 0$ test is inconclusive

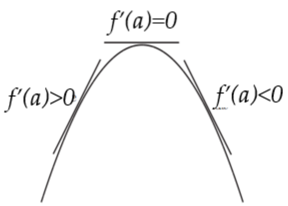
The **relative extrema** are points at which any economic model or function is at a local maximum or minimum. If the function is **strictly concave** (or convex), there is only one maximum (or minimum), called a **global maximum** (or minimum). A differentiable function $f(x)$ is (strictly) concave over an interval if and only if its derivative function $f'(x)$ is (strictly) monotonically decreasing over that interval, that is, the concave function has a decreasing slope. A **monotonic function** is a function that is either always increasing or always decreasing.

Figure 2.1 Minima & Maxima

a. *For minima,* $f''(a) > 0$: convex

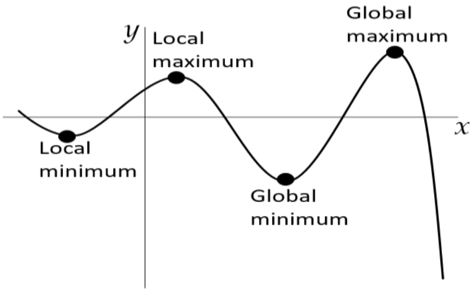


b. *For maxima,* $f''(a) < 0$: concave



The **global maximum** (or minimum) is the single greatest (or smallest) value over the entire function. **Local maximum** (or minimum) is the greatest (or smallest) value within a given range of a function. As shown in Figure 2.2, there is only one global maximum (and one global minimum) but there can be more than one local maximum or minimum. If there is only one local maximum, then it is the global maximum.

Figure 2.2 Global/Local Minima & Maxima



INFLECTION POINTS

As shown in Figure 2.3, an **inflection point** is a point where a function changes concavity (e.g., from being 'concave up' to being "concave down'). Inflection points can only occur when the second derivative is zero or undefined.