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DIFFERENTIATION OF EXPONENTIAL, LOGARITHMIC AND TRIGONOMETRIC FUNCTIONS

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EXPONENTIAL FUNCTIONS

An exponential function is a mathematical function of the form $y = e^p$ where e = 2.71828 and p is a polynomial of degree n.
Example: $y = e^{2x-7}$.

Rules for the differentiation of exponential functions:

$$\frac{d}{dx}(e^u) = \frac{du}{dx} \cdot e^u \qquad\qquad \frac{d}{dx}(a^u) = \frac{du}{dx} \cdot a^u \ln a$$

Special cases: If $y = e^x$, then $dy/dx = e^x$
If $y = a^x$, then $dy/dx = (ln\ a) \cdot a^x$

☑ EXAMPLE 3.1

Differentiate

a) $y = e^{x^2-7}$ b) $y = 5^{\sqrt{x}}$

SOLUTION tips

a) Let $u = x^2 - 7$, then $du/dx = 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(e^u) = \frac{du}{dx} \cdot e^u = 2x \cdot e^{x^2-7}$$

b) Let $u = \sqrt{x} = x^{1/2}$, then $du/dx = \frac{1}{2}x^{-1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}(a^u) = \frac{du}{dx} \cdot a^u \ln a = \frac{1}{2}x^{-1/2} \cdot 5^{\sqrt{x}} \cdot (\ln 5) = \frac{(\ln 5) \cdot 5^{\sqrt{x}}}{2\sqrt{x}}$$

☑ **EXAMPLE 3.2**

Differentiate

a) $y = 2e^x\sqrt{x}$ b) $y = \frac{e^x}{3-e^x}$

SOLUTION**tips**

a) Apply the product rule,

Let $u = 2e^x$, then $du/dx = 2e^x$

Let $v = \sqrt{x}$, then $dv/dx = \frac{1}{2}x^{-1/2}$

$$\frac{dy}{dx} = \sqrt{x} \cdot 2e^x + 2e^x \cdot \frac{1}{2}x^{-1/2} = 2e^x\sqrt{x} + \frac{e^x}{\sqrt{x}}$$

b) Apply the quotient rule,

Let $u = e^x$ and $v = 3 - e^x$

$$\frac{du}{dx} = e^x \quad \text{and} \quad \frac{dv}{dx} = -e^x$$

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2} = \frac{(3 - e^x)e^x - e^x(-e^x)}{(3 - e^x)^2} = \frac{3e^x}{(3 - e^x)^2}$$

LOGARITHMIC FUNCTIONS

Logarithmic functions are the inverse of exponential functions. For example, in the logarithmic function, $y = \log_a x$, y is the logarithm of x to the base of a such that $x = a^y$.

Example: $Log_2 8 = 3$.

Rules for the differentiation of logarithmic functions:

$$\frac{d}{dx}(\ln u) = \frac{du}{dx} \cdot \frac{1}{u} \qquad \frac{d}{dx}(\log_a u) = \frac{du}{dx} \cdot \frac{1}{u \ln a}$$

Special Cases: If $y = \ln x$, then $dy/dx = 1/x$

If $y = log_a x$, then $dy/dx = 1/(x \cdot \ln a)$

☑ **EXAMPLE 3.3**

Differentiate

a) $y = 4 \ln \sqrt{x}$ b) $y = \log_{10}(4x^2 - 3x + 1)$

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