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FURTHER APPLICATIONS OF MULTIVARIABLE CALCULUS

CONTENTS

COMPARATIVE STATICS AND INCOME DETERMINATION MULTIPLIERS

In economics, comparative statics aims to understand how changes in exogenous variables influence the endogenous variables in an economic model. To perform this analysis, partial derivatives are often utilized to calculate the various "multipliers" associated with an income determination model.

The term "multiplier" in this context signifies an economic factor that can trigger changes or shifts in several interconnected economic variables when it is increased or altered. These multipliers play a critical role in understanding the ripple effects of changes in the economy and are central to economic policy analysis.

☑ EXAMPLE 5.1

In a hypothetical economy

$$Y = C + I + G_0 + (X_0 - M_0) \qquad (1)$$

$$C = c_0 + cY \qquad (2)$$

$$I = i_0 + aY \qquad (3)$$

a) Determine the equilibrium level of income.
b) Derive the government, export and import multipliers.

SOLUTION**tips**

a) Substitute (2) and (3) in (1)

$$Y = c_0 + cY + i_0 + aY + G_0 + (X_0 - M_0)$$

Isolate Y

$$Y - cY - aY = c_0 + i_0 + G_0 + (X_0 - M_0)$$

Simplify to obtain the equilibrium level of income

$$\bar{Y} = \frac{1}{1-c-a}(c_0 + i_0 + G_0 + X_0 - M_0) \qquad (4)$$

b) Take the partial derivative of (4) to obtain the multipliers:
The government multiplier is

$$\frac{\partial \bar{Y}}{\partial G_0} = \frac{1}{1-c-a}$$

The export multiplier is

$$\frac{\partial \bar{Y}}{\partial X_0} = \frac{1}{1-c-a}$$

The import multiplier is

$$\frac{\partial \bar{Y}}{\partial M_0} = -\frac{1}{1-c-a}$$

☑ **EXAMPLE 5.2**

Given the model

$$Y = C + I_0 + G_0 \quad (1) \qquad Y_d = Y - T \quad (3)$$
$$C = c_0 + cY_d \quad (2) \qquad T = t_0 + tY \quad (4)$$

a) Derive the equilibrium level of income, the export multiplier and the import multiplier.

b) Given $c = 0.8$, $c_0 = 10$, $t_0 = 5$, $t = 0.3$, $I_0 = 9$ and $G_0 = 7$.
   i. Compute the equilibrium level of income.
   ii. Calculate the effect on $\bar{Y}$ of an increase of 5.5 in t_0.
   iii. By how much should the government change expenditure to achieve a full-employment income of 75?
   iv. Assuming the marginal tax rate is halved, what is the effect on $\bar{Y}$?
   v. By how much should the government change the marginal tax rate to achieve a full-employment income of 60?

SOLUTION**tips**

a) Substitute (2), (3) and (4) in (1)

$$Y = c_0 + c(Y - T) + I_0 + G_0$$

$$Y = c_0 + c\big(Y - (t_0 + tY)\big) + I_0 + G_0$$

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