6

LIMITS, CONTINUITY AND DIFFERENTIABILITY

CONTENTS

LIMITS

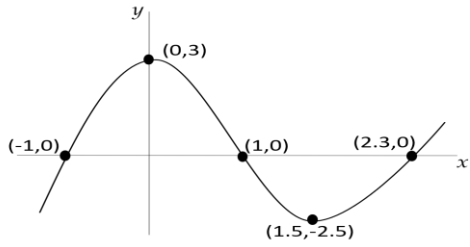
A limit is the value that a function approaches as the input approaches some value. Limits are useful to describe continuity, derivatives, and integrals.
A limit of a function is written as

$$\lim_{x \to a} f(x) = L$$

and is read as "the limit of f of x as x approaches a equals L".

Graphical Limits

Let $f(x)$ be a function whose graph is given below.



The limits are the values that the function approaches as the input approaches the value of x. That is:

$$\lim_{x\to -1} f(x) = 0 \qquad \lim_{x\to 0} f(x) = 3 \qquad \lim_{x\to 1.5} f(x) = -2.5$$

Generally, the limits are the values of the function (if the function is continuous).

CONTINUITY

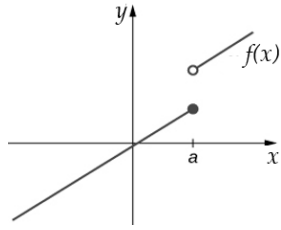
A function $f(x)$ is continuous at $x = a$ if

a) $f(x)$ has a limit as $x \to a$,
b) $f(x)$ is defined at $x = a$, and
c) $\lim_{x\to a} f(x) = f(a)$

A **discontinuous** function has gaps along its graph. If *f(x)* is not continuous at $x = a$, then *f(x)* is said to be discontinuous at this point. The function probably has a hole or breaks.

If a function $f(x)$ is **discontinuous** at $x = a$, then

a) $f(x)$ has a **jump discontinuity** at $x = a$ if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$. The right-hand and left-hand limits of the function are not equal.



b) $f(x)$ has a **removable discontinuity** at $x = a$ if $\lim_{x\to a} f(x)$ exists and this limit is finite. The function is undefined at $x = a$. The graph is unconnected at a point but can be made connected by filling in a single point.