

# 6

## LIMITS, CONTINUITY AND DIFFERENTIABILITY

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### LIMITS

A limit is the value that a function approaches as the input approaches some value. Limits are useful to describe continuity, derivatives, and integrals.

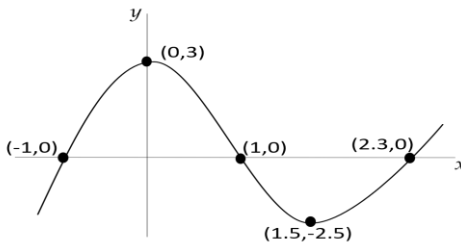
A limit of a function is written as

$$\lim_{x \rightarrow a} f(x) = L$$

and is read as "the limit of  $f$  of  $x$  as  $x$  approaches  $a$  equals  $L$ ".

### Graphical Limits

Let  $f(x)$  be a function whose graph is given below.



The limits are the values that the function approaches as the input approaches the value of  $x$ . That is:

$$\lim_{x \rightarrow -1} f(x) = 0$$

$$\lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 1.5} f(x) = -2.5$$

Generally, the limits are the values of the function (if the function is continuous).

## CONTINUITY

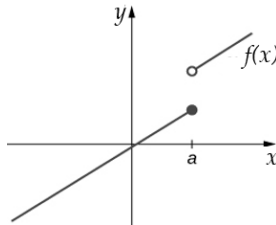
A function  $f(x)$  is continuous at  $x = a$  if

- $f(x)$  has a limit as  $x \rightarrow a$ ,
- $f(x)$  is defined at  $x = a$ , and
- $\lim_{x \rightarrow a} f(x) = f(a)$

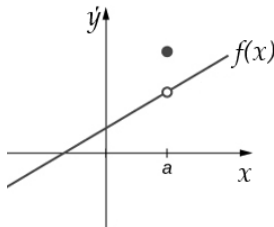
A **discontinuous** function has gaps along its graph. If  $f(x)$  is not continuous at  $x = a$ , then  $f(x)$  is said to be discontinuous at this point. The function probably has a hole or breaks.

If a function  $f(x)$  is **discontinuous** at  $x = a$ , then

- $f(x)$  has a **jump discontinuity** at  $x = a$  if  $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$ . The right-hand and left-hand limits of the function are not equal.



- $f(x)$  has a **removable discontinuity** at  $x = a$  if  $\lim_{x \rightarrow a} f(x)$  exists and this limit is finite. The function is undefined at  $x = a$ . The graph is unconnected at a point but can be made connected by filling in a single point.



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