

7

POLYNOMIALS: REMAINDER & FACTOR THEOREM

CONTENTS

Addition and Subtraction of Polynomials	85
Multiplying Polynomials	86
Using Polynomial Models	87
Polynomial Division	89
Remainder Theorem	92
Factor Theorem	92

A **polynomial** is an algebraic expression of one or more algebraic terms, especially the sum of several terms that contain different powers of the same variable(s) (such as $ax^2 + bx + c$). In other words, a polynomial is a string of mathematical terms. **Monomial** is the polynomial with one term (such as ax^2); **binomial** is the polynomial with two unlike terms (such as $ax^2 + b$), and **trinomial** is the polynomial with three unlike terms (such as $ax^2 + bx + c$).

ADDITION AND SUBTRACTION OF POLYNOMIALS

Adding and subtracting polynomials is simply adding and subtracting their like terms.

☑ EXAMPLE 7.1

Evaluate $(11x^2 - x) + (x^2 + 6x + 1)$.

SOLUTION tips

When adding polynomials, drop the parenthesis and combine like terms.

$$\begin{aligned}(11x^2 - x) + (x^2 + 6x + 1) &= 11x^2 + x^2 - x + 6x + 1 \\ &= 12x^2 + 5x + 1\end{aligned}$$

EXAMPLE 7.2

Evaluate $(x^2 + 3x - 8) - (-x^2 - x + 2)$.

SOLUTIONtips

When subtracting polynomials, distribute the negative first, then combine like terms.

$$\begin{aligned}(x^2 + 3x - 8) - (-x^2 - x + 2) &= x^2 + 3x - 8 + x^2 + x - 2 \\ &= 2x^2 + 4x - 10\end{aligned}$$

MULTIPLYING POLYNOMIALS

Multiplying polynomials involves applying the rules of exponents and the distributive property to simplify polynomials. The general rule is that every term in the first factor has to multiply every term in the other factor. Multiplying polynomials follow four methods: **distribute**, **FOIL**, **rows** and **box**, as shown in Example 7.3.

EXAMPLE 7.3

Simplify $(x + 2y)(3x - 4y)$.

SOLUTIONtips

Multiplying polynomials follow four methods: distribute, FOIL, rows and box.

$$\begin{aligned}\text{Distribute} \quad x(3x - 4y) + 2y(3x - 4y) &= 3x^2 - 4xy + 6xy - 8y^2 \\ &= 3x^2 + 2xy - 8y^2\end{aligned}$$

$$\begin{aligned}\text{FOIL} \quad x(3x) + x(-4y) + 2y(3x) + 2y(-4y) \\ &= 3x^2 - 4xy + 6xy - 8y^2 \\ &= 3x^2 + 2xy - 8y^2\end{aligned}$$

$$\begin{array}{r} \text{Rows} \\ (x + 2y) \\ \times (3x - 4y) \\ \hline -4xy - 8y^2 \\ 3x^2 + 6xy \\ \hline 3x^2 + 2xy - 8y^2 \end{array}$$

Using the FOIL method:

- **F**irst, multiply the first term of each binomial.
- Multiply the **o**utside two terms.
- Multiply the **i**nside two terms.
- Multiply the **l**ast term of each binomial.

The letters of FOIL can help you remember each combination.

Purchase the full book at:

<https://unimath.5profz.com/>

*We donate 0.5% of the book sales every year to charity, forever. When you buy **University Mathematics (I & II)** you are helping orphans and the less privileged.*