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PARTIAL FRACTIONS & BINOMIAL SERIES

CONTENTS

PARTIAL FRACTIONS

Partial fraction decomposition or partial fraction expansion is the process of expressing a rational function (where the numerator and the denominator are both polynomials) as a sum of its initial polynomial fractions. The concept was discovered by both Johann Bernoulli and Gottfried Leibniz independently in 1702.

Form of Algebraic Fraction	*Form of Partial Fractions*
$\frac{px+q}{(x+a)(x+b)}$	$\frac{A}{(x+a)}+\frac{B}{(x+b)}$
$\frac{px+q}{(x+a)^2}$	$\frac{A}{(x+a)}+\frac{B}{(x+b)^2}$
$\frac{px^2+qx+r}{(x+a)(x+b)(x+c)}$	$\frac{A}{(x+a)}+\frac{B}{(x+b)}+\frac{C}{(x+c)}$
$\frac{px^2+qx+r}{(x+a)^2(x+b)}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+b)}$
$\frac{px^2+qx+r}{(x+a)(x^2+bx+c)}$	$\frac{A}{(x+a)}+\frac{Bx+C}{(x^2+bx+c)}$ where x^2+bx+c cannot be factorised.

For an algebraic fraction to be expressed in partial fractions, the numerator must be at least one degree less than the denominator.

☑ EXAMPLE 8.1 TWO LINEAR FACTORS

Express in partial fractions

$$\frac{7x+16}{x^2+2x-8}$$

SOLUTION**tips**

Factorise x^2+2x-8

$$\frac{7x+16}{x^2+2x-8}=\frac{\mathrm{A}}{x+4}+\frac{\mathrm{B}}{x-2}$$

Multiply both sides by $(x+4)(x-2)$

$$7x+16=A(x-2)+B(x+4)$$

Equate the first factor to zero: $(x-2)=0 \quad \rightarrow \quad x=2$

When $x=2$;

$$7(2)+16=A(2-2)+B(2+4)$$

$$14+16=A(0)+6B$$

$$30=6B \quad \rightarrow \quad B=5$$

Equate the second factor to zero: $(x+4)=0 \Rightarrow x=-4$

When $x=-4$;

$$7(-4)+16=A(-4-2)+B(-4+4)$$

$$-28+16=A(-6)+B(0)$$

$$-12=-6A \quad \rightarrow \quad A=2$$

Therefore

$$\frac{7x+16}{x^2+2x-8}=\frac{2}{x+4}+\frac{5}{x-2}$$

☑ EXAMPLE 8.2 REPEATED FACTORS

Express in partial fractions

$$\frac{2x-5}{(x-3)^2}$$

SOLUTION**tips**

$$\frac{2x-5}{(x-3)^2}=\frac{A}{x-3}+\frac{B}{(x-3)^2}$$

Multiply both sides by the common denominator $(x-3)^2$

$$2x-5=A(x-3)+B$$

$$(x-3)=0 \quad \rightarrow \quad x=3$$

$$2(3)-5=A(3-3)+B$$

$$1=A(0)+B \quad \rightarrow \quad B=1$$

Substituting $B=1$ in (i)

$$2x-5=A(x-3)+1$$

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