9

CALCULUS: INTEGRATION

CONTENTS

INTEGRATION

We know from the previous chapters that a function, $y = f(x)$, can be differentiated to obtain the derivative, $\frac{dy}{dx}$. The reverse process of obtaining $f(x)$ from knowledge of its derivative is called integration.

The process of **integration** (or **antidifferentiation**) undoes the process of differentiation. For example, if we differentiate the function $y = x^2$, we obtain $\frac{dy}{dx} = 2x$. Integration reverses this process, and we say that the integral of $2x$ is x^2. There are many functions we can differentiate to give $2x$: $x^2 - 1$, $x^2 + 8$, $x^2 + 0.2$. Because of this, we include an arbitrary constant, c. We say that $2x$ is integrated with respect to x to give $x^2 + c$.

The symbol for integration is $\int$, known as an integral sign. We can therefore write:

$$\int 2x \; dx = x^2 + c$$

dx is a notation which indicates the name of the variable involved. An arbitrary constant of integration, c, is also included. Arbitrary means 'can take any value'.

Based on the results they produce, integrals are divided into two classes: definite and indefinite integrals. A **definite integral** has upper and lower limits on the integrals; it's called definite because, at the end of the solution, we have a definite number which represents the area under the curve $f(x)$ from $x = a$ to $x = b$. On the other hand, an **indefinite integral** is a function; it has no limits and thus have a constant of integration.

Table 9.1 Rules of Integration

	Integrals	*Derivatives*
Basic functions	$\int x^n dx$	$\frac{x^{n+1}}{n+1} + c,\ n \neq -1$
	$\int (ax+b)^n dx$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$
	$\int \frac{1}{x} dx$	$\ln \lvert x \rvert + c$
	$\int \frac{d}{ax+b} dx$	$\frac{d}{a} \ln \lvert ax+b \rvert + c$
Exponential functions	$\int e^{ax} dx$	$\frac{1}{a} e^{ax} + c$
	$\int a^x dx$	$\frac{a^x}{\ln a} + c$
Trigonometric functions	$\int sin\ x\ dx$	$-\ cos\ x + c$
	$\int cos\ x\ \ dx$	$sin\ x + c$
	$\int \tan x\ \ dx$	$-ln\lvert cosx \rvert + c$
Logarithmic functions	$\int \ln x\ \ \ dx$	$x\ ln\ x - x + c$
	$\int log_a x\ dx$	$x\ log_a\ x - \frac{\mathrm{x}}{\ln a} + c$
Rational functions	$\int \frac{1}{1+x^2} dx$	$\arctan x + c = \tan^{-1} x + c$
	$\int \frac{1}{\sqrt{1+x^2}} dx$	$\operatorname{arcsin} x + c = \sin^{-1} x + c$
	$\int \frac{1}{\lvert x \rvert \sqrt{x^2-1}} dx$	$\operatorname{arcsec} x + c = \sec^{-1} x + c$

INDEFINITE INTEGRALS

The result of finding an indefinite integral is usually a function plus a constant of integration. For example:

$$\int x^2\ dx = \frac{x^{2+1}}{2+1} + c = \frac{x^3}{3} + c$$