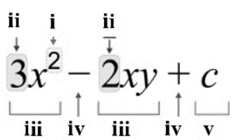
3

ALGEBRA, FACTORIZATION & SURDS

CONTENTS

ALGEBRA

Algebra is a branch of mathematics in which letters and symbols are used to represent numbers and quantities in formulae and equations. The most common letters in use are x and y. An **algebraic expression** is simply a combination of these letters, brackets and other mathematical symbols.



Algebraic expression notation:

i. power (exponent)
ii. coefficient
iii. term
iv. operator
v. constant term

You can work out operations on algebraic expressions (and all mathematical calculations) in the following order:

Brackets 1st	(B)
Indices 2nd	(I)
Division and Multiplication 3rd	(DM)
Addition and Subtraction 4th	(AS)

This is easily remembered using the acronym BIDMAS.

Addition and Subtraction of Like Terms

Like terms are multiples of the same quantity. Examples of like terms are:

ax and bx are multiples of x.
$x^2y, -3x^2y, -5yx^2$ are multiples of x^2y
$abc^2, -2abc^2, habc^2$ are multiples of abc^2

Like terms are added or subtracted to simplify expressions.

Removing Brackets: a(b+c)

To remove brackets is to multiply out.

For example: $3(1+2) = 3 \times 1 + 3 \times 2 = 3 + 6 = 9$
$4(x+y) = 4 \times x + 4 \times y = 4x + 4y$
$a(b+c) = a \times b + a \times c = ab + ac$

Table 3.1 Rules of Algebra

Distributive law: $a(b+c) = ab + ac$
Difference of 2 squares: $(a+b)(a-b) = a^2 - b^2$
Perfect squares: $(a \pm b)^2 = a^2 \pm 2ab + b^2$
Sums of 2 cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of 2 cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Removing Brackets: $(a+b)(c+d)$

$$(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd$$

☑ EXAMPLE 3.1

Simplify

a) $27 \times \left(1 - \frac{1}{3}\right)$ b) $11 - 2(y+3)$ c) $\frac{3a^3b}{12ab^2}$

SOLUTION **tips**

a) Using the distributive law

$$27 \times \left(1 - \frac{1}{3}\right) = 27 \times 1 - 27 \times \frac{1}{3}$$
$$= 27 - 9 = 18$$

b) $11 - 2(y+3) = 11 - 2 \times y - 2 \times 3$
$$= 11 - 2y - 6 = 5 - 2y$$

c) $\frac{3a^3b}{12ab^2} = \frac{3}{12} \times \frac{a^3}{a^1} \times \frac{b^1}{b^2}$
$$= \frac{1}{4}a^{3-1}b^{1-2}$$
$$= \frac{1}{4}a^2b^{-1} = \frac{a^2}{4b}$$

☑ EXAMPLE 3.2

Simplify a) $(a+3)^2$ b) $(3m+4)(3m-4)$

SOLUTION **tips**

a) $(a+3)^2 = (a+3)(a+3) = a(a+3) + 3(a+3)$
$$= a^2 + 3a + 3a + 9 = a^2 + 6a + 9$$

b) Note that $(a+b)(a-b) = a^2 - b^2$
Thus, $(3m+4)(3m-4) = (3m)^2 - 4^2 = 9m^2 - 16$

☑ EXAMPLE 3.3

Simplify

a) $\frac{4}{x-2} \div \frac{7}{2x-4}$ b) $\frac{2}{x+1} + \frac{3}{x-1}$

SOLUTION **tips**

a) $\frac{4}{x-2} \div \frac{7}{2x-4} = \frac{4}{x-2} \times \frac{2x-4}{7}$
$$= \frac{4(2x-4)}{7(x-2)} = \frac{4 \times 2(x-2)}{7(x-2)} = \frac{8}{7}$$

b) $\frac{2}{x+1} + \frac{3}{x-1} = \frac{2(x-1) + 3(x+1)}{x^2-1}$
$$= \frac{2x - 2 + 3x + 3}{x^2-1} = \frac{5x+1}{x^2-1}$$

↘ WORKOUT 3.1

1. Simplify
   a) $-a^2(4 + 3ab - c)$ b) $(x-4)(y+3)$ c) $(a+4)(2a-3b-1)$
2. Simplify
   a) $\frac{-4x+5y}{2} + 3x - 6y$ b) $\frac{3x}{1-x} + 3$ c) $\frac{5(3-x)}{6} + \frac{3(x-5)}{2} + \frac{x}{3}$
   d) $\frac{2x-1}{2x+3} + \frac{1}{3}$ e) $\frac{2}{x-5} + \frac{1}{x+2}$ f) $\frac{3}{3-x} \div \frac{15}{5+x}$

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1. a) $-4a^2 - 3a^2b + a^2c$ b) $xy + 3x - 4y - 12$
   c) $2a^2 - 3ab + 7a - 12b - 4$
2. a) $\frac{2x-7y}{2}$ b) $\frac{3}{1-x}$ c) $x - 5$
   d) $\frac{8x}{3(2x+3)}$ e) $\frac{3x-1}{(x-5)(x+2)}$ f) $\frac{5+x}{5(-x+3)}$

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