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31

# COORDINATE GEOMETRY

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COORDINATE GEOMETRY

Coordinate geometry, also known as analytic geometry, is a branch of geometry where points in the (x, y) plane are defined with the aid of an ordered pair of numbers known as coordinates. Coordinate geometry was developed by the French mathematician René Descartes (1596 – 1650).

DISTANCE BETWEEN TWO POINTS

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
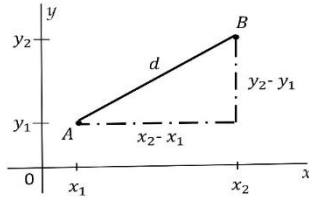
☑   **EXAMPLE 31.1**

Find the distance between the points
   a)   A(-3, −2) and B(6, 8)?   b) D(0, 1) and C(-1, 11)?

SOLUTION **tips**

a) $x_1 = -3, x_2 = 6, y_1 = -2, y_2 = 8$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-3))^2 + (8 - (-2))^2}$$
$$= \sqrt{9^2 + 10^2} = \sqrt{181} = 13.45$$
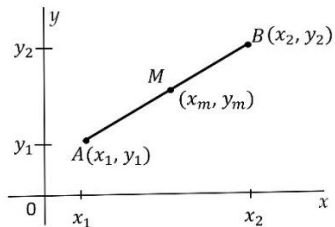b) $x_1 = 0, x_2 = -1, y_1 = 1, y_2 = 11$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - (0))^2 + (11 - (1))^2}$$
$$= \sqrt{(-1)^2 + 10^2} = \sqrt{101} = 10.05$$

MID-POINT OF A LINE SEGMENT

The coordinates of the mid-point M of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$(x_m, y_m) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

☑   **EXAMPLE 31.2**

Find the mid-point of the line segment joining the points A(3, 4) and B(-1, 6).

SOLUTION**tips**

The midpoint is

$$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{3 + (-1)}{2}, \frac{4 + 6}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1, 5)$$

The midpoint is (1, 5).

☑   **EXAMPLE 31.3**

The mid-point M of the line segment joining the point $(-2, 5)$ to the point P is $(2, -1)$. Find the coordinates of the point M.

SOLUTION**tips**

Let the coordinates of the points M be (x_m, y_m). We can then write

$$\left(\frac{x_m - 2}{2}, \frac{y_m + 5}{2}\right) = (2, -1)$$

Equate the x-coordinates

$$\frac{x_m - 2}{2} = 2 \qquad \rightarrow \qquad x_m = 6$$

Equate the y-coordinates

$$\frac{y_m + 5}{2} = -1 \qquad \rightarrow \qquad y_m = -7$$

The coordinates of the point M are $(6, -7)$.

↘   **WORKOUT 31.1**

1.   Find the distance between the points
     a)   A(5, −3) and B(2, -3)   b) A(5/2, 3) and B(0, 7/2)
2.   Find the mid-point of the line segment joining the points
     a)   (6, −6) and (3, 4)   b) (7, 1) and (-1, -1)
3.   The mid-point P of the line segment joining the point (0, -1) to the point P is (3, 1). Find the coordinates of the point P.

ANSWERS RAPID

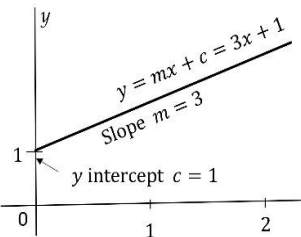1. a) 3   b) √(13/2)   2. a) (9/2, -1) b) (3, 0)   3. (6, 3)

THE GRADIENT/EQUATION OF A STRAIGHT LINE

The equation of a straight line is

$$y = mx + c$$

The gradient of a straight line is a measure of its slope. Given any two points A(x_1, y_1) and B(x_2, y_2) on a line, the gradient of the line is

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

m is the gradient of the line. c is where the line cuts the y-axis. c is called the y-intercept and it has the coordinates (0, c).

For example, the line $y = 3x + 1$ has a gradient of 3 and crosses the y-axis at the point (0, 1).

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