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Discrete & Continuous Probability

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Random Variables: Probability Distributions

A **random variable** (or **stochastic variable**) is described as a variable whose numerical values depend on outcomes of a random phenomenon. A random variable can be either **discrete** or **continuous** depending on the numerical values it assumes. A discrete random variable assumes either a finite number of values or an infinite sequence of values such as 0, 1, 2, ... For example, consider the experiment of users signing in to Instagram. The random variable of interest is x, the number of users signing in during a one-hour period. The possible values of x are the sequence of integers 0, 1, 2.... Thus, x is a discrete random variable assuming one of the values in this sequence.

A **continuous random variable** is a random variable that assumes any numerical value in an interval or collection of intervals. Examples are time, weight, and distance. For example, consider an experiment of monitoring the number of minutes it takes students to complete a test. Suppose the random variable of interest is x, number of minutes it takes students to complete a test. An infinite number of values are possible for x, including values such as 25.10 minutes, 39.912 minutes, 42.8637 minutes, and so on.

Probability distributions describe the likelihood of outcomes in a random event. Discrete probability distributions (e.g., Bernoulli, Binomial, Poisson) have specific values, while continuous probability distributions (e.g., Normal, Exponential, Student's t, Chi-squared) have values within a range.

Discrete Probability Distributions

The **probability distribution** of a random variable describes how probabilities are distributed over the values. If x = *number of successes occurring in the n trials*, x can be 0, 1, 2, 3, . . . , n. x is a *discrete* random variable because the number of values is finite. For a discrete random variable x, the probability distribution is defined by a probability function, denoted by $f(x)$, which provides the probability for each value of the random variable.

Expected value of a discrete random variable

$$\boldsymbol{E(x) = \mu = \sum x f(x)}$$

The variance of a discrete random variable

$$Var(x) = \sigma^2 = \sum (x - u)^2 f(x)$$

The standard deviation of a discrete random variable

$$\sigma = \sqrt{\sigma^2}$$

☑ EXAMPLE 26.1

The following data were collected by counting the number of goals scored by players in the World Cup.

Scores	0	1	2	3	4	5	6
Number of players	11	20	14	15	10	13	17

a) Use the relative frequency approach to construct an empirical discrete probability distribution for the number of goals scored by the players. Show that your probability distribution satisfies the required conditions for a valid discrete probability distribution

b) Compute the probability that (i) a player scored no goals (ii) a player scored not more than two goals (iii) a player scored at least five goals.

SOLUTION**tips**

a) Discrete probability distribution for the number of goals scored:

	x	*f(x)*
0	11	$11/100 = 0.11$
1	20	$20/100 = 0.20$
2	14	$14/100 = 0.14$
3	15	$15/100 = 0.15$
4	10	$10/100 = 0.10$
5	13	$13/100 = 0.13$
6	17	$17/100 = 0.17$
	100	**1**

b) (i) The probability that a player scored no goals

$= f(0) = 0.11$

(ii) The probability that a player scored not more than two goals

$= f(0) + f(1) + f(2)$

$= 0.11 + 0.20 + 0.14 = 0.45$

(iii) The probability that a player scored at least five goals

$= f(5) + f(6)$

$= 0.13 + 0.17 = 0.30$

☑ EXAMPLE 26.2

The demand for pizza at Instant Kitchen varies greatly from day to day. The probability distribution in the following table, based on the past one month of data, shows the company's monthly demand.

Demand for pizza	296	335	464	361	304	210	180
Probability	0.16	0.21	0.16	0.10	0.12	0.15	0.10

What is the expected value of the monthly demand? What are the variance and the standard deviation of the monthly demand.

SOLUTION**tips**

x	$f(x)$	$xf(x)$	$(x-\mu)^2$	$(x-\mu)^2 f(x)$
296	0.16	47.36	325.08	52.01
335	0.21	70.35	439.74	92.35
464	0.16	74.24	22491.00	3598.56

361	0.10	36.10	2206.18	220.62
304	0.12	36.48	100.60	12.07
210	0.15	31.50	10822.24	1623.34
180	0.10	18.00	17964.04	1796.40
Total		**314.03**		**7395.35**