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FUNCTIONS & APPLICATIONS

CONTENTS

FUNCTIONS

A function explains the systematic relationship between two or more variables. For example, $y = f(x)$, if y is a function of x. The function is a rule telling us how to obtain y values from x values. x is the **independent variable** while y is the **dependent variable**[1]. For example, according to economic theory, demand D for a product depends on its price P which can be expressed as a functional relationship $\boldsymbol{D = f(P)}$. In a graphical plot, the independent variable is plotted on the horizontal axis, the dependent variable on the vertical axis.

In a multivariate function, the dependent variable, y, is a function of more than one independent variable. For example, $y = f(x, z)$, if y is a function of the two variables x and z. To find the value of the function y, we substitute values for x and z. In comparative statics analysis, we can hold one variable constant and examine the effect on y of changing the other.

IDENTITIES

An identity explains an enduring relationship between two variables, which are equal by definition. It can show a definitional condition or an equilibrium condition. For example, total profit is the excess of a firm's total revenue, TR, over its total cost, TC:

$$\pi = TR - TC$$

Likewise, saving, S, is the difference between income, Y, and consumption, C:

$$S = Y - C$$

TYPES OF FUNCTIONS

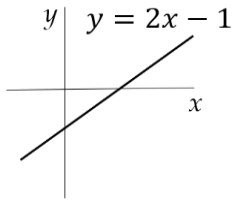
Linear function: $y = a + bx$
Here, b is the slope of the function, and a is the vertical intercept.
Example: $y = 10 + 3x$

[1] In statistical analysis, a dependent variable is a variable that is being studied and measured in response to changes in another variable, known as the independent variable. The independent variable is the variable that is being manipulated or changed to see its effect on the dependent variable.

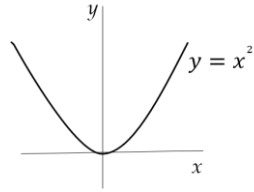
Quadratic function: $y = ax^2 + bx + c$
where a, b and c are constants and $a \neq 0$.
Example: $y = -x^2 + 6x - 1$

Figure 8.1 Graphs of Functions

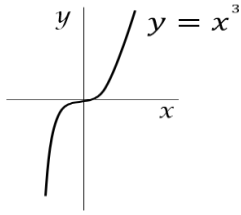
Linear function



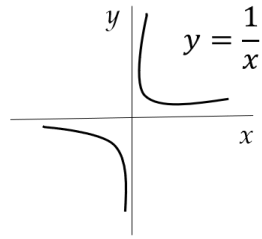
Quadratic function



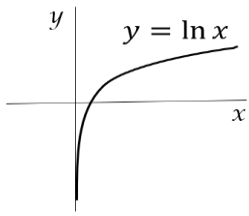
Polynomial function of degree 3



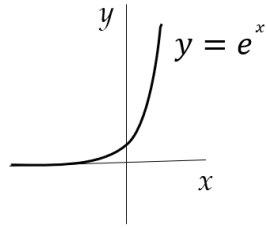
Rational function



Logarithmic function



Exponential function



Polynomial function: $y = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$ where $a_0 \neq 0$ and a_i are constants, n is a non-negative integer and x is a variable.
Example: $y = 2x^4 + x^3 - 4x^2 - x + 1$, of degree $n = 4$.

Logarithmic function: $x = log_a N$
Where x is the logarithm of N to the base of a such that $a^x = N$.
Example: $\log_2 8 = 3$.

Exponential function: $y = e^p$
where $e = 2.71828$ and p is a polynomial of degree n.
Example: $y = e^{2x-7}$.
Logarithmic functions are the inverse of exponential functions: $x = a^y$ is the inverse of $y = \log_a x$.

Rational function: $y = g(x)/h(x)$ where $g(x)$ and $h(x)$ are polynomial functions. *Example*: $y = \frac{x-2}{x+4}$

ECONOMIC FUNCTIONS

For a firm, total revenue, TR, is the price, P, that is paid multiplied by the quantity, Q, sold:

$$TR = P \cdot Q$$