27

HYPOTHESIS TESTING

CONTENTS

HYPOTHESIS TESTING

Hypothesis testing is a statistical method used to make inferences about a population parameter based on a sample of data. The choice of test depends on the type of data and the research question. Common tests include *t*-tests, *z*-tests, chi-square tests, and *F*-tests (ANOVA).

Types of Tests

- *One-tailed test:* Tests if a parameter is greater than or less than a value.
  Example: Testing if a new TikTok feature increases likes. $H_0: \mu \leq 50$, $H_A: \mu > 50$
- *Two-tailed test:* Tests if a parameter is different from a value (either higher or lower).
  Example: Testing if TikTok user engagement changes, without specifying direction. $H_0: \mu = 50, H_A: \mu \neq 50$

NOTE: Think of hypotheses as TikTok features you want to test. Is the new feature better? Set H_0 as the status quo and H_A as the new feature. Collect and analyze data to see if you can reject H_0 and support H_A.
Hypothesis testing helps determine if a new feature makes a real impact. Only one hypothesis can be true at a time.

Errors in Hypothesis Testing

Type I Error (α) occurs when H_0 is true but is rejected. *Example*: Concluding a new TikTok feature increases watch time when it doesn't.
Level of Significance (α) is the probability of making a Type I error, often set at 0.05 or 0.01.
Type II Error (β) occurs when H_0 is false but is accepted. *Example*: Concluding the new TikTok feature does not increase watch time when it does.

Managing Errors
Control Type I Error by setting a low α if the cost of this error is high.
Typically, only Type I error is controlled; hence, statisticians recommend using "do not reject H_0" instead of "accept H_0" to avoid Type II errors.

Steps in hypothesis testing

1. *State the Hypotheses:*
   - *Null Hypothesis* (H_0): A statement that there is no effect or no difference.
   - *Alternative Hypothesis* (H_A): A statement that there is an effect or a difference.

2. *Compute the Appropriate Test Statistic:*
   - Use sample data to calculate the value of the test statistic. This depends on the type of data and the hypothesis. Common tests include t-tests, chi-square tests, ANOVA, etc.
3. *Formulate the Decision Rule:*
   - Determine the critical value(s) from the statistical distribution corresponding to the test statistic and the chosen significance level α. Common values of α are 0.05, 0.01, or 0.10, representing a 5%, 1%, or 10% risk of rejecting the null hypothesis when it is true.
4. *Make a Decision:*
   - Compare the test statistic to the critical value(s):
     - If the test statistic falls into the rejection region, reject H_0
     - Otherwise, do not reject H_0
5. *Draw a Conclusion:*
   - Interpret the results in the context of the research question.

Z-TESTS: LARGE SAMPLE TESTS

The *Central Limit Theorem* states that the sample mean ($\bar{x}$) is approximately normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$. When the sample size is large (n ≥ 30), we can use the Central Limit Theorem to test a population mean. The test statistic is calculated as

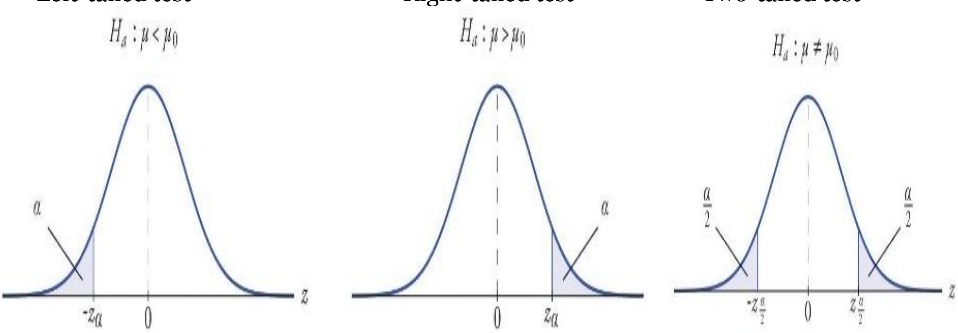
$$\text{If } \sigma \text{ is known: } \boldsymbol{z} = \frac{\bar{\boldsymbol{x}} - \boldsymbol{\mu}}{\boldsymbol{\sigma}/\sqrt{\boldsymbol{n}}}$$

$$\text{If } \sigma \text{ is unknown: } \boldsymbol{z} = \frac{\bar{\boldsymbol{x}} - \boldsymbol{\mu}}{\boldsymbol{s}/\sqrt{\boldsymbol{n}}}$$

$\bar{x}$ = mean, n = sample size, s = population standard deviation, σ = population standard deviation. The test statistic follows the standard normal distribution, which allows us to use standard normal tables to find critical values.

The figure below illustrates the distribution of the standardized test statistic (z) and the corresponding shaded rejection regions for different alternative hypotheses (left-tailed, right-tailed, or two-tailed).

Left-tailed test Right-tailed test Two-tailed test



The critical value (z_α or $-z_\alpha$) is chosen based on the desired significance level (α), which represents the probability of rejecting the null hypothesis when it is true (Type I error).

☑ EXAMPLE 27.1

A social media marketing firm claims that their enhanced strategy has increased the average number of likes per post. Previously, the mean number of likes per post was 1900 with a standard deviation of 120 likes. After implementing the new strategy, a sample of 64 posts is selected and the mean number of likes per post is found to be 1960. Test, at the 1% level of significance, whether the mean number of likes per post has increased due to the new strategy. Assume the number of likes per post follows a normal distribution.

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