4

INDICES, LOGARITHMS & GROWTH

CONTENTS

INDICES

Indices explain how many copies of the base number are multiplied.

$$a^1 = a$$
$$a^2 = a \times a$$
$$a^n = a \times a \times \ldots n \text{ times}$$

For example, 2^5 is a power, where 2 is called the **base** and 5 is called the **index** or **exponent**.

Descartes in 1637 was the first to use this shorthand definition 2^4 for $2 \times 2 \times 2 \times 2$.

Indices and logarithms follow the laws/rules in Table 4.1.

☑ EXAMPLE 4.1

Simplify a) $8^{\frac{2}{3}}$ b) $\sqrt{9^{-3}}$ c) 11^0

SOLUTION**tips**

a) $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$ $\qquad a^{\frac{m}{n}} = \sqrt[n]{a^m}$

b) $\sqrt{9^{-3}} = \left(\sqrt{9}\right)^{-3} = \frac{1}{(3)^3} = \frac{1}{27}$ $\qquad a^{-n} = \frac{1}{a^n}$

c) $11^0 = 1$ $\qquad a^0 = 1$

Tab 4.1 Laws of Indices & Logarithms

	Laws of Indices	*Laws of Logarithm*
Zero Exponents	$a^0 = 1$	$\log_a 1 = 0$
Identity	$a^1 = a$	$\log_a a = 1$
Product	$a^m \bullet a^n = a^{m+n}$	$\log_a(m \bullet n) = \log_a(m) + \log_a(n)$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\log_a \frac{m}{n} = \log_a(m) - \log_a(n)$
Negative Exponents	$a^{-n} = \frac{1}{a^n}$	$\log_a \frac{1}{n} = -\log_a n$
Properties of Equality	If a = b, $a^n = b^n$	$\log_a(a^n) = n$ *or* $a^{\log_a(n)} = n$

Common Base Property of Equality	If $a^m = a^n$, m = n	If $\log_a(m) = \log_a(n)$, then m = n
Power	$(a^m)^n = a^{m \bullet n}$	$\log_a(m^n) = n \bullet \log_a(m)$
Power of a Product	$(a \bullet b)^m = a^m b^m$	
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	
Rational Exponents	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	
Change of Base		$\log_a(m) = \frac{\log_x(m)}{\log_x(a)}$

Note: These properties are true for a>0, b>0, and all values of m and n.

☑ **EXAMPLE 4.2**

Simplify

a) $(16x^6)^{\frac{1}{2}}$ b) $\left(\frac{3}{4}\right)^{-2}$ c) $30x^5y^4 \div 6xy$

SOLUTION tips

a) $(16x^6)^{\frac{1}{2}} = 16^{\frac{1}{2}} \times (x^6)^{\frac{1}{2}}$

$= 4^{2\times\frac{1}{2}} \times x^{6\times\frac{1}{2}}$ $(a^m)^n = a^{m \bullet n}$

$= 4 \times x^3 = 4x^3$

b) $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$ $a^{-n} = \frac{1}{a^n}$

$= \frac{4^2}{3^2} = \frac{16}{9}$

c) $30x^5y^4 \div 6xy$

$= \frac{30x^5y^4}{6xy} = \frac{30}{6} \times \frac{x^5}{x} \times \frac{y^4}{y}$

$= 5x^{5-1}y^{4-1}$ $\frac{a^m}{a^n} = a^{m-n}$

$= 5x^4y^3$

↘ **WORKOUT 4.1**

Simplify

1. $(3y^3)^4$
2. $\frac{p^3 \times p^5}{(2p)^6}$
3. $-\frac{5a^3b - 3ab^2}{3b^2 - 5a^2b}$
4. $\frac{3x^7 \times 2x^4}{5x^6}$
5. $\frac{y^8}{(y^2)^4}$
6. $-\frac{x}{\sqrt{x}}$
7. $\frac{35b^3 + 40b^2}{5b^2}$
8. $\frac{3b^{-3}(b^2 - 1)}{1 - b^{-2}}$
9. $\frac{24}{\sqrt{2}}\pi\left[\sqrt{\frac{1}{8\pi}}\right]^3$

ANSWERS RAPID

1. $81y^{12}$
2. $p^2/64$
3. a
4. $6x^5/5$
5. 1
6. $-\sqrt{x}$
7. $7b + 8$
8. $3/b$
9. $\frac{3}{4\sqrt{\pi}}$

LOGARITHMS

Invented by the Scottish mathematician John Napier, **logarithm** is the power to which a number must be raised in order to get another number. Logarithm is written as "$\log_b x$" and read as "log to base b of x".

A logarithm is a mirror image of an indices. You can convert an exponential equation into an equivalent logarithmic equation and vice versa.

$y = \log_b x$ is equivalent to $b^y = x$.

$2 = \log_{10} 100$ is equivalent to $10^2 = 100$

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