5

LINEAR EQUATIONS & INEQUALITIES

CONTENTS

SOLVING LINEAR EQUATIONS

A **linear equation** is an algebraic equation in which each term has an exponent of one and the graphing of the equation results in a straight line. The case of just one variable is especially important, and often the term linear equation refers to this case.
Examples of linear equations:

$$x + 1 = 0 \qquad 2x + 4 = x - 2 \qquad 1 = \frac{x}{7} + 5$$

Facts to remember about solving an equation:

- To solve an equation, isolate the unknown, which is typically x, on one side of the equation.

$$2x = 1 \quad \rightarrow \quad x = \frac{1}{2}$$

- Shift a term to the other side of an equation by changing its sign.

$$x - 1 = 3 \quad \rightarrow \quad x = 3 + 1 = 4$$
$$x + 4 = 5 \quad \rightarrow \quad x = 5 - 4 = 1$$

- Simplify both side of the equation by removing parentheses and combining like terms:

$$-(x - 9) = 2(3x + 1)$$

Subtraction: $-(x - 9) = -x + 9$
Multiplication: $2(3x + 1) = 6x + 2$

$$-x + 9 = 6x + 2$$

Combine like terms

$$7x = 7 \quad \rightarrow \quad x = 1$$

- Clear fractions by multiplying by the lowest common denominator (LCD is the lowest common multiple of the denominators of several fractions).

$$\frac{1}{x} + 2 = 1 + \frac{4}{3x}$$

The LCD is $3x$. Multiply both sides of the equation by $3x$.

$$3 + 6x = 3x + 4 \quad \rightarrow \quad x = \frac{1}{3}$$

- Decimals can be removed from an equation before solving. Multiply by a power of 10 large enough to make all decimal numbers whole numbers.

$$0.05x + 0.2 = 0.3x - 0.15$$

Multiply both sides by 100.

$$5x + 20 = 30x - 15 \quad \rightarrow \quad x = \frac{7}{5}$$

☑ EXAMPLE 5.1

Solve a) $x - 1 = 0$ b) $-5x + 8 = 3$

SOLUTION **tips**

a) Add 1 to both sides
$x - 1 + 1 = 0 + 1$
$x = 1$
NOTE: Whatever you do to one side, you must do to the other!

b) Subtract 8 from both sides
$-5x + 8 - 8 = 3 - 8$
$-5x = -5$
Divide both sides by -5
$\frac{-5x}{-5} = \frac{-5}{-5}$
$x = 1$

☑ EXAMPLE 5.2

Solve $2 + 0.04x = 1.29x - 0.5$

SOLUTION **tips**

Decimals can be removed from an equation before solving. Multiply by a power of 10 large enough to make all decimal numbers whole numbers.
Multiply by 100: $200 + 4x = 129x - 50$
Collect like terms: $125x = 250$
Divide both sides by 125: $x = 2$

☑ EXAMPLE 5.3

Solve

$$\frac{11}{x-1} + \frac{3}{4} = 1$$

SOLUTION **tips**

The Least Common Denominator (LCD) is the smallest multiple that two or more denominators share. Clear fractions by multiplying by the LCD: $4(x - 1)$

$$11(4) + 3(x - 1) = 4(x - 1)$$

Expand: $44 + 3x - 3 = 4x - 4$
Collect like terms: $3x - 4x = -4 - 44 + 3$
Simplify: $-x = -45$ Divide both sides by -1: $x = 45$

☑ EXAMPLE 5.4

Solve

$$\frac{x + \frac{1}{2}}{24x - 8} - \frac{4x + 1}{3x - 1} + 6 = \frac{1}{4}$$

SOLUTION **tips**

Clear fractions by multiplying by the LCD: $24x - 8$

$$x + \frac{1}{2} - 8(4x + 1) + 6(24x - 8) = 6x - 2$$

Clear the remaining fraction by multiplying both sides by 2

$$2x + 1 - 16(4x + 1) + 12(24x - 8) = 12x - 4$$

Expand and simplify: $226x - 111 = 12x - 4$
Collect like terms: $214x = 107$
Divide both sides by 214: $x = \frac{1}{2}$

↘ WORKOUT 5.1

1. Solve a) $9x - 3 = 42$ b) $-1 + (6/11)\,x = x - (71/11)$
2. Solve
   a) $\frac{x}{2x-6} - \frac{1}{x-3} = 1$
   b) $\frac{5}{2+x} + \frac{12}{5} = \frac{41}{15}$
   c) $-\frac{7}{10} = \frac{7}{10}x + 6$
   d) $\frac{x}{12+3x} + \frac{x-1}{x+4} + \frac{1}{3} = 2$

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