# 2

# LOGIC

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**Logic** is the study of the methods and principles of correct reasoning.
Logic is a fundamental tool used across various fields:

- *Lawyers and Business People:* To construct valid arguments, analyze legal contracts, and solve complicated problems.
- *Computer Programmers:* To design software.
- *Engineers:* To design electronic circuits.
- *Mathematicians:* To solve problems and construct mathematical proofs.

When people say 'logic', they mean either propositional logic or first-order predicate logic. The simplest and most abstract logic is propositional logic.

## PROPOSITIONAL LOGIC

**Propositional logic**, also known as **sentential logic** or **statement logic**, is a branch of logic which deals with propositions and their interrelationships. A **proposition** is a statement which is either true or false. The term proposition is sometimes used synonymously with statement. There are two logical values: true and false, denoted by T and F, respectively.

Examples of propositions:

- The car is on.
- The book is open.
- Sir Sigma is the president.
- $2026 + 10 = 2036$

The following are non-propositions:

- Are you going out today?
- 1+2 (This is a mathematical expression, not a statement.)
- Five miles from the university

To determine whether any given statement is a proposition, prefix it with: *"It is true that ..."* Check whether the result makes grammatical sense. Rather than write out propositions in full, they are abbreviated using propositional variables; they are denoted by small letters (*p, q, r, ...*).

Further examples of propositions:

*p:* = Man is mortal.
The logical value of the proposition $p$ is true.

*q:* = If $x = -1$ then $x^2 + 1 = 0$.
The logical value of the proposition $q$ is false.

## TRUTH TABLE

The **truth table** is used to show whether a proposition is logically valid. It has one column for each proposition (for example, $p$ and $q$), and one final column showing all of the possible results of the logical operation that the table represents. Each row contains one possible configuration of the propositions (e.g., $p$ = true, $q$ = false), and the result of the operation for those values.

**Table 2.1** Truth Table

| *Relation* | $p$<br>$q$ | *T*<br>*T* | *T*<br>*F* | *F*<br>*T* | *F*<br>*F* |
|---|---|---|---|---|---|
| conjunction | $p \wedge q$ | T | F | F | F |
| disjunction | $p \vee q$ | T | T | T | F |
| implication | $p \Rightarrow q$ | T | F | T | T |
| equivalence | $p \Leftrightarrow q$ | T | F | F | T |

## CONNECTIVES AND OPERATIONS BETWEEN PROPOSITIONS

**Negation: (denoted by ~, called "not")**

The negation of a proposition $p$ is defined by

$$\sim p := \begin{cases} \text{true}, & \text{if } p \text{ is false} \\ \text{false}, & \text{if } p \text{ is true} \end{cases}$$

That is, if $p$ is true, then $\sim p$ is false; if $p$ is false, $\sim p$ is true.

*Example:* $p$: The President is coming today.
$\sim p$: The President is not coming today.

The truth table for negation looks like this:

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

**Conjunction: (denoted by ∧, called "and")**

The conjunction of propositions $p$ and $q$ is defined by

$$p \wedge q := \begin{cases} \text{true}, & \text{if } p \text{ and } q \text{ are both true} \\ \text{false}, & \text{if at least one of } p \text{ and } q \text{ is false} \end{cases}$$

That is, it is true when, and only when, both $p$ and $q$ are true. If either $p$ or $q$ is false, or if both are false, $p \wedge q$ is false.

*Example:* $p$: Today is Sunday.
$q$: It is raining today.
$p \wedge q$: Today is Sunday and it is raining today.
This proposition is true only on rainy Sundays and is false on any other rainy day or on Sundays when it does not rain.

The truth table for conjunction looks like this:

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Disjunction: (denoted by ∨, called "or")**

The disjunction of propositions $p$ and $q$ is defined by

$$p \vee q := \begin{cases} \text{true}, & \text{if at least one of } p \text{ and } q \text{ is true} \\ \text{false}, & \text{if } p \text{ and } q \text{ are both false} \end{cases}$$

That is, it is true when at least one of $p$ or $q$ is true and is false only when both $p$ and $q$ are false.

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