# 10

# PERMUTATION & COMBINATION

CONTENTS



## FACTORIAL

The factorial of $n$ (denoted by $n!$) is the product of all positive integers less than or equal to $n$. For example, $2!$ is $2 \times 1$.

**Table 10.1** Factorials

| | | |
|---|---|---|
| $1! =$ | $1$ | 1 |
| $2! =$ | $2 \cdot 1$ | 2 |
| $3! =$ | $3 \cdot 2 \cdot 1$ | 6 |
| $4! =$ | $4 \cdot 3 \cdot 2 \cdot 1$ | 24 |
| $5! =$ | $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ | 120 |
| $6! =$ | $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ | 720 |
| $7! =$ | $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ | 5,040 |
| $8! =$ | $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ | 40,320 |
| $9! =$ | $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ | 362,880 |

$n!$ is $n$ multiplied by all of the positive integers smaller than $n$.

## COUNTING PRINCIPLES

There are some basic counting techniques useful in determining the number of different ways of arranging or selecting objects. The two basic counting principles are:

MULTIPLICATION PRINCIPLE: If there are $n$ events and if the first event occurs in $k_1$ ways, the second event occurs in $k_2$ ways after the first event has occurred, the third event occurs in $k_3$ ways after the second event has occurred, etcetera, then all the n events occurs in $k_1 \times k_2 \times \ldots \times k_{n-1} \times k_n$ ways.

ADDITION PRINCIPLE*:* If an event $P$ occurs in $m$ ways and another event $Q$ occurs in $n$ ways, and suppose that both do not occur together, then $P$ or $Q$ can occur in $m + n$ ways.

The application of these principles:

- In selecting clothing combinations, if Joe has 3 pants and 2 shirts, there are 3 choices of pants and 2 choices of shirts. Utilizing the multiplication principle, which states that the total number of outcomes of sequential events is the product of individual event possibilities, we can compute the total number of combinations: $3 \times 2 = 6$ possible outfits.

- Similarly, in forming words from given letters, if the word is "REST," and repetition is not allowed, we can choose the first letter from the available letters (R, E, S, T) in 4 ways. After choosing the first letter, there remain 3 letters to choose from for the second position. For the third position, there remain 2 choices, and for the fourth position, there is only 1 choice left. The number of 4-letter words is $4 \times 3 \times 2 \times 1 = 24$.

### ☑ EXAMPLE 10.1

How many multiples of 5 are there from 10 to 85?

SOLUTION**tips**

The multiples are 2-digit integers between 10 and 85.
The first digit can be any one of the 8 digits: 1,2,3,4,5,6,7,8.
The second digit can be 0 or 5: these can be chosen in 2 ways.
Thus, there are

$$2 \times 8 = 16 \text{ multiples of 5 from 10 to 85}$$

### ☑ EXAMPLE 10.2

Akon can travel from A to B by 2 buses, from B to C by 3 buses, from C to D by 4 buses and from D to E by 2 buses. In how many ways can he travel from A to E?

SOLUTION**tips**

The bus from A to B can be selected in 2 ways.
The bus from B to C can be selected in 3 ways.
The bus from C to D can be selected in 4 ways.
The bus from D to E can be selected in 2 ways.
So, Akon can travel from A to E in $2 \times 3 \times 4 \times 2$ ways $=$ 48 ways.

## PERMUTATIONS

A permutation is an arrangement of objects in a definite **order**. Here, order is important.
Consider the scenario: forming 3-letter words from "BUMPER." The outcome varies based on whether repetition of letters is allowed or not.
**Without repetition:** If repetition of letters is not allowed, each letter can only be used once in a word. This results in permutations, where the order matters. In this case, there are 6 options for the first letter, 5 options for the second letter (as one letter has been used), and 4 options for the third letter. Thus, the total number of permutations is $6 \times 5 \times 4 = 120$.
**With repetition:** If repetition of letters is allowed, each letter can be used multiple times in a word. This results in more permutations compared to the scenario without repetition. In this case, there are still 6 options for the first letter, but for the second and third letters, we can choose from all 6 letters (including repetitions). Thus, the total number of permutations is $6 \times 6 \times 6 = 216$.
In summary, allowing repetition increases the number of permutations because it introduces more options for each position in the word.
There are basically three types of permutation:

- Permutations when repetition is allowed
- Permutations of $n$ different objects
- Permutations where some items are identical

### I. Permutations when repetition is allowed

Permutations with repetition occur when there are items that can be repeated in a permutation. The formula is simply:

$$n^r$$

*Purchase the full book at:*

https://unimath.5profz.com/

*We donate 0.5% of the book sales every year to charity, forever. When you buy* **University Mathematics (I & II)** *you are helping orphans and the less privileged.*