6

QUADRATIC EQUATIONS

CONTENTS

STANDARD FORM

A **quadratic equation** is an equation of the second degree; it contains at least one term that is squared. The standard form is $ax^2 + bx + c = 0$, where a, b, and c are numerical coefficients; x is an unknown variable.

The standard form of a quadratic equation:

$$\boldsymbol{ax^2 + bx + c = 0}$$

Where $a, b,$ and c are integers and $a \neq 0$

Quadratic equations commonly appear in various areas of mathematics, physics, engineering, and other sciences. They are used to model a wide range of phenomena, including projectile motion, optimization problems, and electrical circuits.

The form $ax^2 + c = 0$, where $b = 0$.

This is the simplest quadratic equation to solve. You can easily solve this using the Square Root Property.

☑ **EXAMPLE 6.1**

Solve (a) $4x^2 - 16 = 0$ (b) $8x^2 - 18 = 0$

SOLUTION **tips**

a) Divide both sides by 4

$$\frac{4x^2}{4} = \frac{16}{4}$$

$$x^2 = 4$$

Take the square root of both sides

$$\sqrt{x^2} = \sqrt{4}$$

Thus

$$x = \pm 2$$

b) Divide both sides by 8

$$\frac{8x^2}{8} = \frac{18}{8}$$

$$x^2 = \frac{9}{4}$$

Take the square root of both sides

$$\sqrt{x^2} = \sqrt{\frac{9}{4}} \quad \rightarrow \quad x = \pm \frac{3}{2}$$

The form $ax^2 + bx = 0$, where $c = 0$.

This can be solved by factoring.

☑ **EXAMPLE 6.2**

Solve (a) $12x^2 - 48x = 0$ (b) $20x^2 - 220x = 0$

SOLUTION**tips**

a) Factorise the LHS

$$12x(x-4) = 0$$
$$12x = 0 \quad \text{or} \quad x-4=0$$
$$x = 0 \quad \text{or} \quad x = 4$$

b) Factorise the LHS

$$20x(x-11) = 0$$
$$20x = 0 \quad \text{or} \quad x-11=0$$
$$x = 0 \quad \text{or} \quad x = 11$$

The form $ax^2 + bx + c = 0$.

To solve these types of quadratic equations, you have three options, (1) to use Factoring; (2) to Complete the Square, or (3) to use the Quadratic Formula.

FACTORING

A quadratic equation can easily be solved by factoring if it is factorable. To factorise a quadratic equation, look for two numbers a and b whose sum is $(a+b)$ and whose products is ab.

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

☑ **EXAMPLE 6.3**

Solve $x^2 - 6x + 5 = 0$

SOLUTION**tips**

Look for two numbers whose product is *coefficient of* $x^2 \times 5 = 5$ and whose sum is -6.

The two numbers are -1 and -5 because $(-1) + (-5) = -6$ and $(-1) \times (-5) = 5$

Replace -6x with $-1x - 5x$

$$x^2 - 1x - 5x + 5 = 0$$

Factorise

$$x(x-1) - 5(x-1) = 0 \quad \rightarrow \quad (x-1)(x-5) = 0$$
$$x-1=0 \quad x-5=0 \quad \rightarrow \quad x = 1 \quad \text{or} \quad x = 5$$

☑ **EXAMPLE 6.4**

Solve $2x^2 - 7x - 4 = 0$

SOLUTION**tips**

Look for two numbers whose product is *coefficient of* $x^2 \times -4 = -8$ and whose sum is -7.

The two numbers are -8 and 1 because $(-8) + 1 = -7$ and $(-8) \times 1 = -8$

Replace -7x with $-8x + 1x$

$$2x^2 - 8x + 1x - 4 = 0$$

Factorise

$$2x(x-4) + 1(x-4) = 0 \quad \rightarrow \quad (2x+1)(x-4) = 0$$
$$2x+1=0 \quad x-4=0 \quad \rightarrow \quad x = -1/2 \quad \text{or} \quad x = 4$$

COMPLETING THE SQUARE

Completing the square is a useful method for solving quadratic equations and is often used when factoring is not immediately applicable.

The form $ax^2 + bx + c = 0$ where $a = 1$

☑ **EXAMPLE 6.5**

Solve $x^2 + 8x + 15 = 0$