9

SEQUENCES & SERIES

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SEQUENCES

Sequence is a set of numbers arranged in order by some particular rule.
For example,

$$1, 3, 5, 7, 9, \ldots \qquad \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$$

Consider the sequence $a_1, a_2, a_3, \ldots, a_n$.
a_1 is the first term, a_2 is the second term, a_3 is the third and so on.
A **finite sequence** has a limited number of terms e.g., 2, 4, 6, 8, 10.
An **infinite sequence** has unlimited number of terms e.g., $1, 3, 5, 7, 9, \ldots$

ARITHMETIC PROGRESSION

An **arithmetic progression** (AP) is a sequence where the difference in consecutive terms is constant. The sequence 5, 7, 9, 11, 13, 17 is an example of an AP. The sequence starts with 5 and thereafter each term is 2 more than the previous one. This difference of 2 is called the **common difference**. The sum of an arithmetic sequence is called an **arithmetic series**.

Table 9.1 Formulae for Arithmetic Progression

n^{th} term of an AP	$a_n = a_1 + (n-1)d$
Sum of n terms in an AP	$S_n = \frac{n}{2}[2a_1 + (n-1)d]$
Sum of n terms in an AP when the first and last terms are known	$S_n = \frac{n}{2}(a_1 + a_n)$

a_1 = the first term, a_n = the last term, d = the common difference.

☑ EXAMPLE 9.1

In an AP, the first term is 15 and the common difference is – 2. Find
a) the 6^{th} term b) the sum of the first 10 terms.

SOLUTION tips

a) $a_1 = 15, n = 6, d = -2$
$a_n = a_1 + (n-1)d$
$a_6 = 15 + (6-1)(-2)$
$= 5$

b) $n = 10$
$S_n = \frac{n}{2}[2a_1 + (n-1)d]$
$S_{10} = \frac{10}{2}[2(15) + (10-1)(-2)] = 60$

☑ EXAMPLE 9.2

An arithmetic progression has 10 as its first term. Also, its 23rd term is $\frac{6}{25}$ of its 99th term. Find the common difference.

SOLUTION tips

$$a_1 = 10$$
$$a_n = a_1 + (n-1)d$$

When $n = 99$: $a_{99} = 10 + (99-1)d = 10 + 98d$
When $n = 23$: $a_{23} = 10 + (23-1)d = 10 + 22d$
But

$$a_{23} = \frac{6}{25}(a_{49})$$
$$10 + 22d = \frac{6}{25}(10 + 98d)$$

Simplify

$$25(10 + 22d) = 6(10 + 98d) \quad \rightarrow \quad d = 5$$

☑ EXAMPLE 9.3

Find the sum of the series $7 + 16 + 25 + \cdots$ to 15 terms.

SOLUTION tips

Here $a_1 = 7$, $d = 16 - 7 = 9$, $n = 15$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$
$$S_{10} = \frac{15}{2}[2(7) + (15-1)9] = 1050$$

☑ EXAMPLE 9.4

The income of Ms Noe is $3,000 in the first year and she receives an increase of $100 per year. Find the total amount she receives in 15 years.

SOLUTION tips

Here, we have

$a_1 = 3000$, $d = 100$, $n = 15$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$
$$S_{15} = \frac{15}{2}[2(3000) + (15-1)100] = 55{,}500$$

☑ EXAMPLE 9.5

Makanaki repays her mortgage of $662,500 by $1,000 in the first monthly instalment and then increases the payment by $500 in every instalment. How many instalments will it take her to pay off the mortgage?

SOLUTION tips

By the given conditions, we have
Here $a_1 = 1000$, $d = 500$, $S_n = 662500$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$
$$662500 = \frac{n}{2}[2(1000) + (n-1)(500)]$$

Multiply both sides by 2

$$1325000 = n[2(1000) + (n-1)(500)]$$

Simplify

$$500n^2 + 1500n - 1325000 = 0$$

Divide both sides by 500

$$n^2 + 3n - 2650 = 0$$