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SET THEORY

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SETS

Georg Cantor, a German mathematician, is widely credited with the development of set theory. A set is a collection of items or **elements**. A set can be defined by listing the elements of the set, enclosed in curly brackets as follows: {1, 2, 3, 4}. Some examples of sets:

Natural numbers: N = {1, 2, 3, . . .}
Whole numbers: W = {0, 1, 2, 3, . . .}
Integers: Z = {. . . , -3, -2, -1, 0, 1, 2, 3, . . .}

Table 1.1 shows some of the various symbols used in set notation.

There are two methods of representing sets:

- roster form (listing elements enclosed in braces) and
- set-builder form (describing elements based on a common property).

Examples:

Roster form	Set-Builder Form
X = {1, 2, 3, 4, 5}	$X = \{x : x$ is a natural number and $x < 6\}$
X = {1, -2}.	$X = \{x : x$ is an integer and $x^2 + x - 2 = 0\}$
X= {1, 4, 9, 16, 25, ...}	$X = \{x : x$ is the square of a natural number}
X = {P, R, I, N, C, E, L}	$X = \{x : x$ is a letter of the word PRINCIPLE}.

Read the braces as "the set of all" and the colon as "such that".

Empty Set: An empty set, denoted by φ or {}, contains no elements.
Example: A set of natural numbers between 1 and 2 is empty: $\{x : 1 < x < 2\}$.

Finite and Infinite Sets: Finite sets have a specific count of elements, while infinite sets continue indefinitely.
Example: The set of days in a week (finite) versus the set of natural numbers (infinite).

Equal Sets: Sets are equal if they contain the same elements.
Example: Sets with the same elements, like {1, 2, 3} and {2, 2, 1, 3, 3}, are considered equal. Duplicates in sets do not affect the equality.

Subsets: A set A is a subset of set B if every element of A is also in B, denoted as $A \subset B$.
Example: A = {1, 2}, B = {1, 2, 3}. $A \subset B$ because all elements of A are in B.

Power Set: The power set of a set A contains all possible subsets of A.
Example: If A = {1, 2}, then P(A) = {φ, {1}, {2}, {1, 2}}.

Table 1.1 Table of Set Notations

Symbol	*Symbol Name*	*Meaning*	*Example*
{ }	Set	a collection of elements	A = {1, 2, 3, 6} B = {3, 6}
|	such that	so that	A = {x : x∈ℝ, x<0}
A∪B	Union	elements that belong to set A or set B	A ∪ B = {1, 2, 3, 6}
A∩B	intersection	elements that belong to set A and set B	A ∩ B = {3}
A⊂B	proper subset	A is a subset of B	{1, 2} ⊂ {1, 2, 3}
A⊄B	not subset	A is not a subset of B	{-1, 0} ⊄ {1, 2, 3}
2^A	power set	all subsets of A	
P(A)	power set	all subsets of A	
A=B	equality	A and B have the same members	A = {11, 12, 13} B = {11, 12, 13} A = B
A^c	complement	all the elements that do not belong to set A	
a ∈ A	element of	set membership	A = {1, 2, 3}, 1 ∈ A
x ∉ A	not element of	no set membership	A = {1, 2, 3}, -2 ∉ A
{}, Ø	empty set		A = {}, A = Ø
U	universal set	set of all possible values	
N	natural numbers	N = {0, 1, 2, 3,...}	0 ∈ N
Z	integer numbers set	Z = {...-2, -1, 0, 1, 2,...}	-6 ∈ Z
Q	rational numbers set	Q = {x | x=a/b, a,b ∈ Z}	2/6 ∈ Q
R	real numbers set	R = {x | -∞ < x <∞}	6.343434 ∈ R

Universal Set: The universal set is the foundational set encompassing all elements relevant to a particular context or problem, denoted by U or ξ.
Example: In human population studies, the universal set consists of all people in the world.

SET OPERATIONS

Union of sets: (denoted by ∪, called "union")
The union of two sets contains all the elements in either sets. The symbol for union is ⋃. Formally $x \in A \cup B$ if $x \in A$ *or* $x \in B$ (or both).

i.e. $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Intersection of sets: (denoted by ∩, called "intersection")
The intersection of two sets contains only the elements that are in both sets. The symbol for intersection is ⋂. More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$.

i.e. $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Complement of a set: (denoted by A^c, called "A complement")
The complement of a set A contains everything not in set A. The symbol for A complement is A^c. It contains all the elements in the universal set that are not in A.

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