7

SIMULTANEOUS EQUATIONS

CONTENTS

SYSTEM OF EQUATIONS IN 2 VARIABLES

Simultaneous equations refer to a set of two or more algebraic equations that involve the same variables and are solved at the same time. Consider the following example:

$x + y = 1 \qquad x - y = 5$

For this set of equations, there is a single combination of values of x and y that satisfies both: $x = 3,\ y = -2$.

Several algebraic techniques exist to solve simultaneous equations. Perhaps the easiest to understand are the elimination, substitution and graphical methods.

Elimination Method

STEPS:
- Make the coefficients of one variable equal.
- Add or subtract the equations to eliminate a variable.
- Solve for the remaining variable.
- Substitute back into an original equation.

☑ EXAMPLE 7.1

Solve $4x - 3y = -2$ and $x + 3y = 7$ simultaneously.

SOLUTION **tips**

Add the two equations together to eliminate the y term.

$$\begin{array}{rl} 4x - 3y = & -2 \\ x + 3y = & 7 \quad \text{Add} \\ \hline 5x \qquad = & 5 \end{array}$$

Divide both sides by 5: $x = 1$

To find the value of y, substitute $x = 1$ into one of the original equations.

$x + 3y = 7 \quad \rightarrow \quad 1 + 3y = 7$

$3y = 7 - 1 = 6 \quad \rightarrow \quad y = \frac{6}{3} = 2$

So $x = 1$ and $y = 2$

CHECK:

You can substitute the values of x and y into both equations to check your answers.

Equation (i): $4\,(1) - 3(2) = -2$ YES

Equation (ii): $1 + 3(2) = 7$ YES

☑ EXAMPLE 7.2

Solve the equations

$$x + \frac{3}{2}y = 1 \qquad \frac{5}{4}x + y = 3$$

SOLUTION **tips**

To eliminate the y term, multiply the first equation by 8 and the second equation by 12; then subtract the first equation from the second.

$$x + \frac{3}{2}y = 1 \xrightarrow{\times 8} 8x + 12y = 8 \quad \text{Subtract}$$

$$\frac{5}{4}x + y = 3 \xrightarrow{\times 12} \underline{15x + 12y = 36}$$

$$7x = 28$$

Divide both sides by 7: $x = 4$

To find the value of y, substitute $x = 4$ into one of the original equations.

$$\frac{5}{4}x + y = 3 \quad \rightarrow \quad \frac{5}{4}(4) + y = 3 \quad \rightarrow \quad y = 3 - 5 = -2$$

So $x = 4$ and $y = -2$

☑ EXAMPLE 7.3

Solve the following system of equations: $-100x + 3y = 12$ and $5x - 5y = -20$.

SOLUTION **tips**

To eliminate the y term:

$$-100x + 3y = 12 \xrightarrow{\times 5} -500x + 15y = 60$$

$$5x - 5y = -20 \xrightarrow{\times 3} \underline{15x - 15y = -60} \quad \text{Add}$$

$$-485x = 0$$

Thus, $x = 0$.

Substitute $x = 0$ in one of the original equations:

$$-100x + 3y = 12 \quad \rightarrow \quad -100(0) + 3y = 12 \quad \rightarrow \quad y = \frac{12}{3} = 4$$

So, $x = 0, y = 4$.

Substitution Method

STEPS:

- Solve one equation for a variable.
- Substitute into the other equation.
- Solve the resulting equation.
- Back-substitute into the first equation.

☑ EXAMPLE 7.4

Solve the simultaneous equations $0.3x - 0.4y = 0.5$ and $0.7x - 1.1y = 1.0$

SOLUTION **tips**

Rearrange the first equation.

$$x = \frac{0.5 + 0.4y}{0.3} \qquad \text{(i)}$$

Substitute equation (i) in the second equation.

$$0.7\left(\frac{0.5 + 0.4y}{0.3}\right) - 1.1y = 1.0$$

Multiply through by 0.3 and simplify.

$$0.35 + 0.28y - 0.33y = 0.3$$

$$-0.05y = -0.05 \quad \rightarrow \quad y = 1$$

To find the value of x, substitute $y = 1$ into equation (i).

$$x = \frac{0.5 + 0.4(1)}{0.3} = \frac{0.9}{0.3} = 3$$

So $x = 3$ and $y = 1$