33

TRIGONOMETRIC FUNCTIONS

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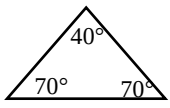
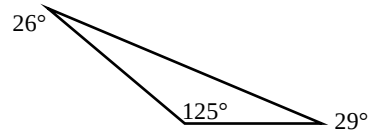
TRIANGLE

A **triangle** is a polygon with three sides and three angles. It is the basic shape in trigonometry. **Trigonometry** is from the Greek words "trigonon" which means triangle and "metron" which means measure. It is a branch of mathematics that studies relationships involving lengths and angles of triangles.
The angles of a triangle always add up to 180°.

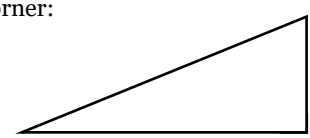


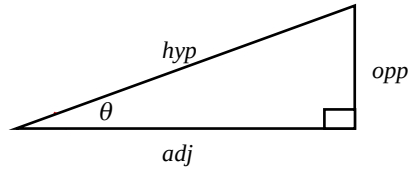
40°+ 40°+70° = 180°



26°+ 29°+125° = 180°

The triangle of most interest is the **right-angled triangle**. This is a triangle in which one of the angles is 90°. The right angle is shown by the little box in the corner:





THE PYTHAGOREAN THEOREM

The sides of the right triangle are:

Opp: the side **opposite** the acute angle θ
Adj: the side **adjacent** to the acute angle θ
Hyp: the **hypotenuse** of the right triangle

Pythagoras' theorem states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

$$\mathbf{hyp^2 = adj^2 + opp^2}$$

Sine	**SOH**	→	$\sin\theta = \frac{\textbf{O}pp}{\textbf{H}yp}$
Cosine	**CAH**	→	$\cos\theta = \frac{\textbf{A}dj}{\textbf{H}yp}$
Tangent	**TOA**	→	$\tan\theta = \frac{\textbf{O}pp}{\textbf{A}dj}$

There are three basic trigonometric ratios: sine, cosine, and tangent.
You can use the mnemonic SOH CAH TOA to remember the trigonometric ratios.

☑ EXAMPLE 33.1

In (a) Find x, $\sin\theta$, $\cos\theta$ and $\tan\theta$. In (b) Find x and y.

SOLUTION**tips**

(a)

$$\text{Hyp}^2 = \text{Adj}^2 + \text{Opp}^2$$
$$x^2 = 4^2 + 3^2 = 25$$

Take the square root: $x = 5$

SOH → $\sin\theta = \text{Opp/Hyp} = 3/5$

CAH → $\cos\theta = \text{Adj/Hyp} = 4/5$

TOA → $\tan\theta = \text{Opp/Adj} = 3/4$

(b)

To find the opposite side, given the hypotenuse, use:

$$\sin 51 = \frac{\text{Opp}}{\text{Hyp}} = \frac{x}{11}$$
$$x = 11\sin 51° = 8.55$$

To find the adjacent side, given the hypotenuse and the opposite side, use the Pythagoras' theorem:

$$\text{Hyp}^2 = \text{Adj}^2 + \text{Opp}^2$$
$$11^2 = y^2 + 8.55^2$$

Isolate y^2

$$y^2 = 11^2 - 8.55^2 = 47.8975$$
$$y = \sqrt{47.8975} = 6.92$$

☑ EXAMPLE 33.2

Given $\cos\theta = 3/5$, find the other trigonometric ratios of angle θ.

SOLUTION**tips**

First draw a triangle: so,

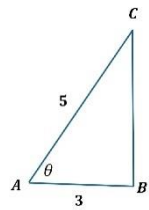
$$\text{Cos}\,\theta = \frac{AB}{AC} = \frac{3}{5}$$

By the Pythagoras Theorem:

$$AB^2 = 5^2 - 3^2 = 16 \quad \rightarrow \quad AB = 4$$

Thus,

$$\sin\theta = \frac{\text{BC}}{\text{AC}} = \frac{4}{5} \qquad \tan\theta = \frac{\text{BC}}{\text{AB}} = \frac{4}{3}$$



☑ EXAMPLE 33.3

A 9 m ladder rests against a wall and makes an angle of 68° with the ground. What is the distance between the base of the ladder and the wall?