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VECTORS

CONTENTS

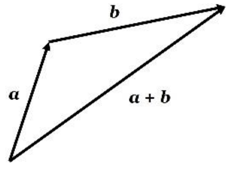
Vectors are quantities possessing magnitude and direction. Examples are forces, displacements, velocities, forces, etc. Vectors can be represented by lines with arrows indicating the direction. A quantity with magnitude but no direction is called a **scalar**.
Vectors can be represented geometrically as directed line segments. The length of the line segment represents the vector's magnitude, and the arrowhead indicates its direction.
Algebraically, vectors are often represented as a list of coordinates. For example, in two dimensions, a vector can be written as $v = (v_x, v_y)$ and in three dimensions, as $v = (v_x, v_y, v_z)$.

VECTOR ADDITION

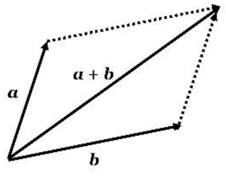
Vectors are added by joining them end to end. Adding vectors can be done using the triangle law or parallelogram law, both of which yield the same result.

Graphical Addition:

Triangle rule:



Parallelogram rule:



Trigonometry can be used to find the numerical values of the length (magnitude) and the angle (direction) of a vector.

Algebraic Addition:

Given two vectors represented in their component forms:

$$a = (a_1, a_2, a_3) \qquad b = (b_1, b_2, b_3)$$

The sum of the two vectors $a + b$ is given by:

$$a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

CARTESIAN COMPONENTS OF A VECTOR

The Cartesian unit vectors have magnitudes of $|\bar{\imath}| = |\bar{\jmath}| = 1$
Any vector $\bar{P}$ can be represented in Cartesian coordinates:

$$\bar{P} = P_x * \bar{\imath} + P_y * \bar{\jmath}$$

Where $P_x = |\bar{P}| * \cos\Theta$; $P_y = |\bar{P}| * \sin\Theta = |\bar{P}| * \cos(90 - \Theta)$

Then

$$|\bar{P}| = \sqrt{P_x^2 + P_y^2} \quad \text{and} \quad \Theta = \tan^{-1}\left({P_y}/{P_x}\right)$$

Given two vectors $\bar{P} = P_x * \bar{\imath} + P_y * \bar{\jmath}$ and $\bar{Q} = Q_x * \bar{\imath} + Q_y * \bar{\jmath}$

$$\bar{P} + \bar{Q} = P_x * \bar{\imath} + P_y * \bar{\jmath} + Q_x * \bar{\imath} + Q_y * \bar{\jmath}$$
$$= (P_x + Q_x) * \bar{\imath} + (P_y + Q_y) * \bar{\jmath}$$

The result is similar for more than two forces.

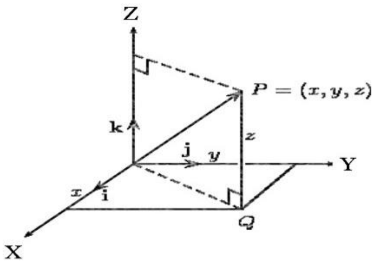
For three-dimensional vectors, the Cartesian unit vectors are:

$$|\bar{\imath}| = |\bar{\jmath}| = |\bar{k}| = 1$$

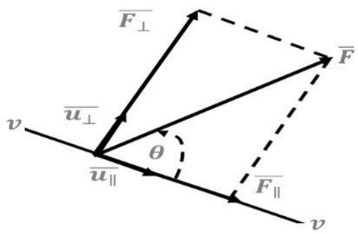
Given a vector $\bar{P} = P_x\bar{\imath} + P_y\bar{\jmath} + P_z\bar{k}$

$$|\bar{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

Three-dimensional vector



Vector parallel and perpendicular to a line



Components of a vector parallel and perpendicular to a line

The parallel component of a vector is defined as:

$$F_\parallel = \mathrm{F} \cdot \cos(\theta) = \bar{F} \cdot \overline{u_\parallel} \qquad \overline{F_\parallel} = F_\parallel \cdot \overline{u_\parallel} = (\bar{F} \cdot \overline{u_\parallel}) \cdot \overline{u_\parallel}$$

The perpendicular component of a vector is defined as:

$$\overline{F_\perp} = \bar{F} - \overline{F_\parallel} \qquad F_\perp = \sqrt{F^2 - F_\parallel^2}$$

THE SCALAR PRODUCT OF TWO VECTORS

The scalar product of two vectors $\bar{A}$ and $\bar{B}$ is written as $\bar{A} \cdot \bar{B}$ and defined as

$$\boxed{\bar{\boldsymbol{A}} \cdot \bar{\boldsymbol{B}} = |\bar{\boldsymbol{A}}| \cdot |\bar{\boldsymbol{B}}| \cdot \boldsymbol{cos(\theta)}}$$

where θ is the angle between vectors $\bar{A}$ and $\bar{B}$, with their tails joined.

NOTE: When two vectors a and b are perpendicular, θ = 90° and cos θ = 0.
Therefore, a.b = 0

From the scalar definition, it is obvious that for Cartesian unit vectors:

$$\bar{\imath} \cdot \bar{\imath} = \bar{\jmath} \cdot \bar{\jmath} = \bar{k} \cdot \bar{k} = 1 \cdot 1 \cdot \cos(0) = 1$$
$$\bar{\imath} \cdot \bar{\jmath} = \bar{\imath} \cdot \bar{k} = \bar{\jmath} \cdot \bar{k} = 1 \cdot 1 \cdot \cos(90°) = 0$$

$\overrightarrow{\mathrm{AB}} = \binom{4}{3}$ means to go from A to B. It is called a **column vector**.

The magnitude or **modulus** of the vector $\overrightarrow{\mathrm{AB}}$ is represented by the length as follows:

$$|\overrightarrow{\mathrm{AB}}|^2 = 4^2 + 3^2 = 25 \qquad |\overrightarrow{\mathrm{AB}}| = \sqrt{25} = 5$$

If given $\overrightarrow{AB}$ and $\overrightarrow{BC}$, then $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

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